EXEMPLO —— REPRESENTAÇÃO DE PLANTAS COM PÓLOS MÚLTIPLOS NA FORMA CANÔNICA MODAL:

$$G(s) = \frac{s+1}{s^2+4s+4}$$

$$F_{cc} = \begin{bmatrix} -4 & -4 \\ 1 & 0 \end{bmatrix} \qquad G_{cc} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad H_{cc} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$|\lambda I - F_{cc}| = \begin{vmatrix} \lambda+4 & 4 \\ -1 & \lambda \end{vmatrix} = \lambda^2 + 4\lambda + 4 = 0$$

$\lambda = -2$ (DUPLO)

AUTOVETOR $(F_{cc} - \lambda I)\, t_1 = 0$

$$\begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} t_{11} \\ t_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$t_{21} = 1 \longrightarrow t_{11} = -2 \longrightarrow t_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

AUTOVETOR GENERALIZADO(*)

$$(F_{cc} - \lambda I)\, t_2 = t_1$$

$$\begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} t_{12} \\ t_{22} \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$t_{22} = 1 \longrightarrow t_{12} = -1 \longrightarrow t_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix} \quad ; \quad T^{-1} = \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix}$$

$$F_{cm} = \underbrace{\begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -4 & -4 \\ 1 & 0 \end{bmatrix}}_{\begin{bmatrix} 3 & 4 \\ -2 & -4 \end{bmatrix}} \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}$$

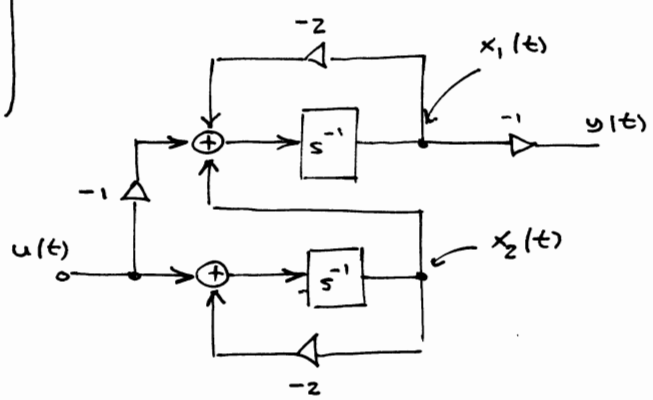
$$G_{cm} = \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$H_{cm} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

DIAGRAMA DE BLOCOS:



EXPANSÃO EM FRAÇÕES PARCIAIS

$$G(s) = \frac{A_1}{(s+2)^2} + \frac{A_2}{(s+2)} = \frac{-1}{(s+2)^2} + \frac{1}{s+2}$$

$$A_1 = \left. s+1 \right|_{s=-2} = -1$$

$$A_2 = \left. \frac{d}{ds}(s+1) \right|_{s=-2} = 1$$

(*) R. HORN AND C. JOHNSON, TOPICS IN MATRIX ANALYSIS, CAMBRIDGE UNIV. PRESS, NEW YORK, 1991.

EXPANSÃO EM FRAÇÕES PARCIAIS — CASO GERAL:

$$G(s) = \frac{N(s)}{(s+a)(s+b)^m} = \frac{A}{s+a} + \frac{B_1}{(s+b)^m} + \frac{B_2}{(s+b)^{m-1}} + \dots + \frac{B_m}{s+b}$$

$$A = \left.\frac{N(s)}{(s+b)^m}\right|_{s=-a}$$

$$B_1 = \left.\frac{N(s)}{(s+a)}\right|_{s=-b}$$

$$B_2 = \left.\frac{d}{ds}\,\frac{N(s)}{(s+a)}\right|_{s=-b}$$

$$\vdots$$

$$B_m = \frac{1}{(m-1)!}\cdot\frac{d^{(m-1)}}{ds^{(m-1)}}\left[\frac{N(s)}{(s+a)}\right]\Bigg|_{s=-b}$$

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