Universidade Federal do Rio de Janeiro

Escola Politécnica

DATA: 27/11/2009

GRAUS: 1 2 3 4 5

Aluno: PROVA PARCIAL #2 — GABARITO

Disciplina: CONTROLE LINEAR II-A

Turma: EEL760 2009/2

Professor: JOSÉ GABRIEL R.C. GOMES

QUESTÃO #1:

a) $$\frac{D(s)}{s} = \frac{s+1}{s(s+2)} = \frac{1/2}{s} + \frac{1/2}{s+2} \quad \therefore \quad \boxed{u(t) = \frac{1}{2}\left(1+e^{-2t}\right)u(t)}$$

b) $$u(k) = \frac{1}{2}\left(1+e^{-2Tk}\right)u(k)$$

$$U(z) = \frac{1}{2}\left(\frac{z}{z-1} + \frac{z}{z-e^{-2T}}\right) \Rightarrow D(z) = \frac{1}{2}\left(1 + \frac{z-1}{z-e^{-2T}}\right) = \boxed{\frac{z-\left(\frac{1+e^{-2T}}{2}\right)}{z-e^{-2T}}}$$

c) $$U(z) = \frac{z}{z-1}\cdot\frac{z-a}{z-b} \Rightarrow \frac{U(z)}{z} = \frac{\frac{1-a}{1-b}}{z-1} + \frac{\frac{b-a}{b-1}}{z-b} = \frac{1/2}{z-1} + \frac{1/2}{z-e^{-2T}}$$

$$\frac{1-\left(\frac{1+e^{-2T}}{2}\right)}{1-e^{-2T}} = \frac{1}{2} \qquad \frac{e^{-2T}-\left(\frac{1+e^{-2T}}{2}\right)}{e^{-2T}-1} = \frac{1}{2}$$

$$\boxed{u(k) = (1/2)\left(1+e^{-2Tk}\right)u(k)}$$ (JÁ SABÍAMOS ISTO, DO ITEM (b))

d) BILINEAR:

$$D(z) = \frac{\frac{2}{T}\left(\frac{z-1}{z+1}\right)+1}{\frac{2}{T}\left(\frac{z-1}{z+1}\right)+2} = \frac{2z-2+Tz+T}{2z-2+2Tz+2T} = \boxed{\frac{(T+2)z+T-2}{(2T+2)z+2T-2}}$$

$$U(z) = \frac{z}{z-1}\cdot\frac{\left(\frac{T+2}{2T+2}\right)z + \frac{T-2}{2T+2}}{z+\left(\frac{2T-2}{2T+2}\right)} = \frac{z}{z-1}\cdot\frac{az+b}{z+c}$$

$$\frac{U(z)}{z} = \frac{\frac{a+b}{c+1}}{z-1} + \frac{\frac{ac-b}{c+1}}{z+c}$$

$$a+b = \frac{T}{T+1}\ ; \quad c+1 = \frac{2T}{T+1}\ ; \quad ac-b = \frac{(T+2)(T-1)2}{4(T+1)^2} - \frac{(T-2)(T+1)2}{4(T+1)^2}$$

$$\frac{ac-b}{c+1} = \frac{1}{2(T+1)} \longleftarrow ac-b = \frac{2(\cancel{T^2}+T-\cancel{2}-\cancel{T^2}+T+\cancel{2})}{4(T+1)^2} = \frac{T}{(T+1)^2}$$

$$\boxed{u(k) = \left(\frac{1}{2} + \frac{1}{2(T+1)}\left(\frac{2-2T}{2+2T}\right)^k\right)u(k)}$$

QUESTÃO #2:

a) $F=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$; $G=\begin{bmatrix}1\\ 0\end{bmatrix}$; $H=[0 \ 1]$;

$$e^{FT}=\mathcal{L}^{-1}\left[(sI-F)^{-1}\right]\Big|_{t=T}=\mathcal{L}^{-1}\left[\begin{bmatrix}s & 0\\ -1 & s\end{bmatrix}^{-1}\right]\Big|_{t=T}=\mathcal{L}^{-1}\left[\begin{bmatrix}\frac{1}{s} & 0\\ \frac{1}{s^2} & \frac{1}{s}\end{bmatrix}\right]\Big|_{t=T}$$

$$\boxed{e^{FT}=\begin{bmatrix}1 & 0\\ T & 1\end{bmatrix}};\ \Gamma=\int_0^T\begin{bmatrix}1\\ \sigma\end{bmatrix}d\sigma=\boxed{\begin{bmatrix}T\\ T^2/2\end{bmatrix}}$$

$$\left(\underline{\text{OBS.:}}\ G(z)=[0 \ 1]\begin{bmatrix}z-1 & 0\\ -T & z-1\end{bmatrix}^{-1}\begin{bmatrix}T\\ T^2/2\end{bmatrix}=[0 \ 1]\frac{\begin{bmatrix}z-1 & 0\\ T & z-1\end{bmatrix}}{(z-1)^2}\begin{bmatrix}T\\ T^2/2\end{bmatrix}=\frac{T^2(z+1)}{2(z-1)^2}\right.$$

b) $$\alpha_c(z)=z^2-(e^{s_1T}+e^{s_1^*T})z+e^{T(s_1+s_1^*)}=z^2+(-2Re(e^{s_1T}))z+e^{2TRe(s_1)}$$

$$\begin{bmatrix}1 & 0\\ \cancel{\phi}T & 1\end{bmatrix}-\begin{bmatrix}T\\ T^2/2\end{bmatrix}[k_1 \ k_2]=\begin{bmatrix}1-Tk_1 & -Tk_2\\ T-\frac{T^2}{2}k_1 & 1-\frac{T^2}{2}k_2\end{bmatrix}$$

$$\begin{vmatrix}z+Tk_1-1 & Tk_2\\ \frac{T^2}{2}k_1-T & z+\frac{T^2k_2}{2}-1\end{vmatrix}=z^2+\left(Tk_1+\frac{T^2k_2}{2}-2\right)z-Tk_1+\frac{T^2k_2}{2}+1$$

$$Tk_1+\frac{T^2k_2}{2}=2-2Re(e^{s_1T}) \implies \boxed{k_1=\frac{3-2Re(e^{s_1T})-e^{2TRe(s_1)}}{2T}}$$

$$-Tk_1+\frac{T^2k_2}{2}=-1+e^{2TRe(s_1)} \implies \boxed{k_2=\frac{1+e^{2TRe(s_1)}-2Re(e^{s_1T})}{T^2}}$$

c) $$\begin{bmatrix}1 & 0\\ T & 1\end{bmatrix}-\begin{bmatrix}\ell_1\\ \ell_2\end{bmatrix}[0 \ 1]=\begin{bmatrix}1 & -\ell_1\\ T & 1-\ell_2\end{bmatrix}$$

$$\begin{vmatrix}z-1 & \ell_1\\ -T & z+\ell_2-1\end{vmatrix}=z^2+(\ell_2-2)z+\ell_1T-\ell_2+1$$

$$\ell_1T+\ell_2=-1+e^{2TRe(s_2)}$$

$$\boxed{\ell_2=2-2Re(e^{s_2T})} \implies \boxed{\ell_1=\frac{-3+e^{2TRe(s_2)}+2Re(e^{s_2T})}{T}}$$

d) $t_p=\frac{\pi}{\omega_d}=\frac{3.14}{10}=\cancel{31.4}\ 0.314 \implies$ EM $\boxed{32}$ AMOSTRAS.

$T=0.01$

QUESTÃO #3:

a) $$\phi-LH\phi=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\begin{bmatrix}\ell_1\\ \ell_2\end{bmatrix}[1 \ 1]=\begin{bmatrix}1-\ell_1 & -\ell_1\\ 1-\ell_2 & 1-\ell_2\end{bmatrix}\cancel{\ }$$

$$|zI-\phi+LH\phi| = \begin{vmatrix} z+\ell_1-1 & \ell_1 \\ \ell_2-1 & z+\ell_2-1 \end{vmatrix} = z^2+(\ell_1+\ell_2-2)z+(\cancel{-\ell_1}-\ell_2+1+\cancel{\ell_1})$$

$$\ell_1+\ell_2=2$$

$$\cancel{1-\ell_1=0} \quad 1-\ell_2=0 \implies \ell_1=1, \quad \ell_2=1$$

$$\boxed{L_a=\begin{bmatrix}1\\1\end{bmatrix}}$$

b) $\phi-LH\phi-\Gamma K+LH\Gamma K=\begin{bmatrix}-2 & -2\\ 0 & 0\end{bmatrix}$

$$\hat{x}(k+1)=\begin{bmatrix}-2 & -2\\ 0 & 0\end{bmatrix}\hat{x}(k)+\begin{bmatrix}1\\1\end{bmatrix}y(k+1)$$

$$u(k)=-[2 \ \ 1]\,\hat{x}(k)$$

$$D(z)=-[2 \ \ 1]\begin{bmatrix}z+2 & 2\\ 0 & z\end{bmatrix}^{-1}\begin{bmatrix}1\\1\end{bmatrix}z=-[2 \ \ 1]\frac{\begin{bmatrix}z & -2\\ 0 & z+2\end{bmatrix}}{z(z+2)}\begin{bmatrix}1\\1\end{bmatrix}z$$

$$D(z)=-\frac{[2z \ \ z-2]}{z^2+2z}\begin{bmatrix}1\\1\end{bmatrix}z \implies \boxed{D(z)=\frac{-3z+2}{z+2}}$$

c) $G(z)=[0 \ \ 1]\begin{bmatrix}z-1 & 0\\ -1 & z-1\end{bmatrix}^{-1}\begin{bmatrix}\cancel{X}1\\ X0\end{bmatrix}=[0 \ \ 1]\frac{\begin{bmatrix}z-1 & 0\\ 1 & z-1\end{bmatrix}}{(z-1)^2}\begin{bmatrix}1\\0\end{bmatrix}=\frac{1}{(z-1)^2}$

$$\frac{G}{1-GD}=\frac{\frac{1}{(z-1)^2}}{1+\frac{3z-2}{\underbrace{(z-1)^2}_{z^2-2z+1}(z+2)}}=\boxed{\frac{z+2}{z^3}} \qquad \text{GANHO DC} = 2.$$

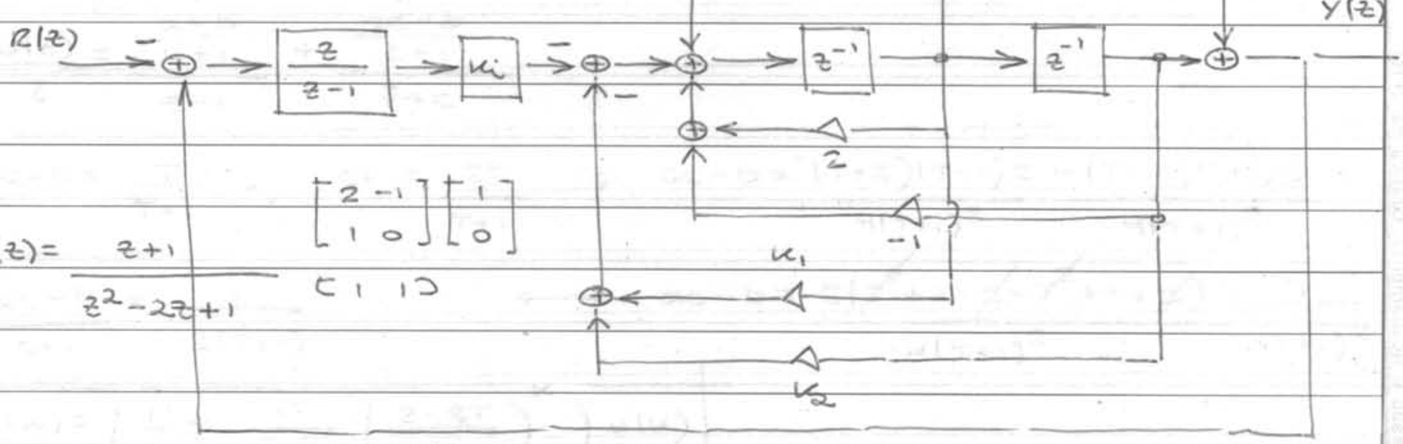
$\implies$

| | | | | | |
|---|---|---|---|---|---|
| | 1 | -2 | 1 | | |
| 2 | 1 | | | | 1 |
| | 2 | 1 | | | 0 |
| | | 2 | 1 | | -3 |
| | | | 2 | 1 | 2 |

d) $\boxed{\bar{N}=1/2}$

QUESTÃO # 4:

a)




$$\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$$

$$G(z)=\frac{z+1}{z^2-2z+1} \qquad [1 \ \ 1]$$

b) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [k_i \;\; k_1 \;\; k_2] = \begin{bmatrix} 1 & 1 & 1 \\ -k_i & 2-k_1 & -1-k_2 \\ 0 & 1 & 0 \end{bmatrix}$

$$\begin{vmatrix} z-1 & -1 & -1 \\ k_i & z+k_1-2 & k_2+1 \\ 0 & -1 & z \end{vmatrix} = z^3 + z^2(k_1-2-1) + (2-k_1)z + k_i + (k_2+1)z + (-k_2-1) + k_i z$$

$$= z^3 + (k_1-3)z^2 + (3-k_1+k_2+k_i)z + k_i - k_2 - 1$$

$\boxed{k_1 = 3}$ $\quad k_2 + k_i = 0 \quad \Rightarrow \quad \boxed{k_2 = -1/2} \quad \boxed{k_i = +1/2}$

$k_2 - k_i = -1$

c)

$$\frac{Y(z)}{R(z)} = [0 \;\; 1 \;\; 1] \begin{bmatrix} z-1 & -1 & -1 \\ 0.5 & z+1 & 0.5 \\ 0 & -1 & z \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \frac{(-1)\times}{z^3} \left[ \begin{vmatrix} 0.5 & 0.5 \\ 0 & z \end{vmatrix} (-1) + \begin{vmatrix} 0.5 & z+1 \\ 0 & -1 \end{vmatrix} \right]$$

$$\frac{Y(z)}{R(z)} = \frac{(-1)(-0.5z-0.5)}{z^3} = \frac{0.5(z+1)}{z^3} \qquad \lim_{z \to 1} \frac{Y(z)}{R(z)} = \boxed{1}$$

$$\frac{Y(z)}{W(z)} = [0 \;\; 1 \;\; 1] \begin{bmatrix} z-1 & -1 & -1 \\ 0.5 & z+1 & 0.5 \\ 0 & -1 & z \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \frac{\begin{vmatrix} z-1 & -1 \\ 0 & z \end{vmatrix} + (-1) \begin{vmatrix} z-1 & -1 \\ 0 & -1 \end{vmatrix}}{z^3}$$

$$\frac{Y(z)}{W(z)} = \frac{z^2-1}{z^3} \qquad \lim_{z \to 1} \frac{Y(z)}{W(z)} = \boxed{0}$$