PROVA PARCIAL #2 — 2009/1 — GABARITO

~~QUESTÃO #4:~~ QUESTÃO #1:

a) $D(s) = \dfrac{s+\overset{3}{\cancel{1}}}{s^2}$

$$D(z) = \frac{\frac{2}{T}\left(\frac{z-1}{z+1}\right)+\overset{3}{\cancel{1}}}{\left(\frac{2}{T}\frac{(z-1)}{(z+1)}\right)^2} = \frac{2(z-1)T(z+1)+3T^2(z+1)^2}{4(z-1)^2}$$

$$D(z) = \frac{(3T^2+2T)z^2 + \overset{6}{\cancel{2}}T^2z + 3T^2 - 2T}{4z^2-8z+4}$$

b) $\dfrac{D(s)}{s} = \dfrac{s+\overset{3}{\cancel{1}}}{s^3} = \dfrac{1}{s^2}+\dfrac{\overset{3}{\cancel{1}}}{s^3}$

$$y(t) = \left(t + 3\frac{t^2}{2}\right)u(t)$$

$$y(k) = \left(kT + 3\frac{(kT)^2}{2}\right)u(t) = Tku(k) + 3\frac{T^2}{2}k^2u(k)$$

$$Y(z) = T\frac{z}{(z-1)^2} + 3\frac{T^2}{2}\cdot\frac{z(z+1)}{(z-1)^3}$$

$$D(z) = \frac{z-1}{z}Y(z) = T\cdot\frac{1}{z-1} + 3\frac{T^2}{2}\cdot\frac{(z+1)}{(z-1)^2}$$

$$= \frac{T(z-1)+3\frac{T^2}{2}(z+1)}{(z-1)^2} = \frac{z\left(3\frac{T^2}{2}+T\right)+3\frac{T^2}{2}-T}{z^2-2z+1}$$

c) $D(s)=\dfrac{s+\overset{3}{\cancel{1}}}{s^2} \longrightarrow x' = \begin{bmatrix}0&0\\1&0\end{bmatrix}x + \begin{bmatrix}1\\0\end{bmatrix}u$

$$y = [1\ \ \overset{3}{\cancel{1}}]\,x$$

$$\Phi = e^{FT} = \mathcal{L}^{-1}\left((sI-F)^{-1}\right)\Big|_{t=T} = \mathcal{L}^{-1}\left[\begin{bmatrix}s&0\\-1&s\end{bmatrix}^{-1}\right]\Big|_{t=T} = \mathcal{L}^{-1}\begin{bmatrix}\frac{1}{s}&0\\\frac{1}{s^2}&\frac{1}{s}\end{bmatrix}\Big|_{t=T} = \begin{bmatrix}1&0\\T&1\end{bmatrix}$$

$$\Gamma = \int_0^T e^{F\sigma}\overset{G}{\cancel{\gamma}}\,d\sigma = \int_0^T \begin{pmatrix}1\\\sigma\end{pmatrix}d\sigma = \begin{bmatrix}T\\T^2/2\end{bmatrix}$$

$$D(z) = [1\ \ \overset{3}{\cancel{1}}]\underbrace{\begin{bmatrix}z-1&0\\-T&z-1\end{bmatrix}^{-1}}_{\frac{\begin{bmatrix}z-1&0\\T&z-1\end{bmatrix}}{(z-1)^2}}\begin{bmatrix}T\\T^2/2\end{bmatrix} = \frac{\left[z+(3T-1)\ \ 3(z-1)\right]\begin{bmatrix}T\\T^2/2\end{bmatrix}}{(z-1)^2}$$

$$D(z) = \frac{\left(3\frac{T^2}{2}+T\right)z + \left(3\frac{T^2}{2}-T\right)}{z^2-2z+1}$$

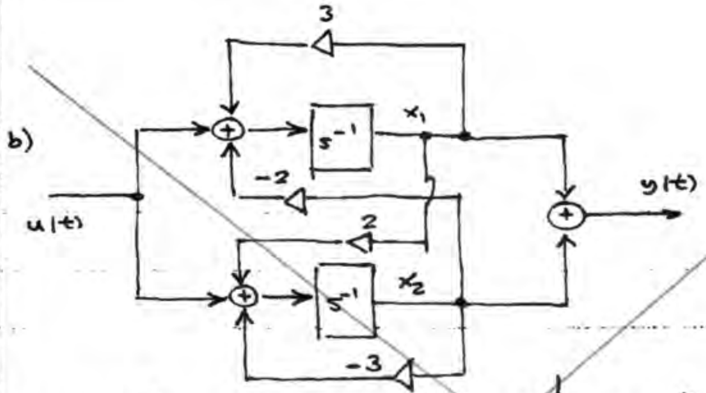
d) (k) ENVIAR $u(k)$ P/ CONVERSOR D/A

AMOSTRAR $y(k)$

$u(k+1) = 2u(k) - u(k-1) + \left(\frac{3T^2}{2}+T\right)y(k) + \left(\frac{3T^2}{2}-T\right)y(k-1)$

TEMPO DE ESPERA

b)



c) $\mathcal{C} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ CONTROLÁVEL ($\det \mathcal{C} = -2$) | $\mathcal{O} = \begin{bmatrix} 1 & 1 \\ 5 & -5 \end{bmatrix}$ OBSERVÁVEL ($\det \mathcal{O} = -10$)

d) $Y(s) = [1 \;\; 1]\begin{bmatrix} s-3 & 2 \\ -2 & s+3 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \dfrac{[1 \;\; 1]\begin{bmatrix} s+3 & -2 \\ 2 & s-3 \end{bmatrix}}{s^2-5}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$Y(s) = \frac{10}{s^2-5} = \frac{-\sqrt{5}}{s+\sqrt{5}} + \frac{\sqrt{5}}{s-\sqrt{5}}$$

$y(t) = \left(\sqrt{5}\,e^{\sqrt{5}t} - \sqrt{5}\,e^{-\sqrt{5}t}\right)u(t)$ (O SISTEMA É INSTÁVEL).

QUESTÃO #2:

~~QUESTÃO #5:~~ d) ZEROS DE $\frac{Y(z)}{R(z)}$ SÃO $1 \pm \sqrt{2}j$ (IGUAIS AOS PÓLOS DO COMPENSADOR)

a) $\phi - \Gamma H - \Gamma K = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} -2 \\ 1 \end{bmatrix}[1 \;\; 0] - \begin{bmatrix} 0 \\ -1 \end{bmatrix}[3 \;\; 2] = \begin{bmatrix} 0 & -1 \\ 3 & 2 \end{bmatrix}$

$D(z) = -[3 \;\; 2]\begin{bmatrix} z & 1 \\ -3 & z-2 \end{bmatrix}^{-1}\begin{bmatrix} -2 \\ 1 \end{bmatrix} = -\dfrac{[3 \;\; 2]\begin{bmatrix} z-2 & -1 \\ 3 & z \end{bmatrix}}{z^2-2z+3}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$$D(z) = \frac{-[3z \;\; 2z-3]\begin{bmatrix} -2 \\ 1 \end{bmatrix}}{z^2-2z+3} = \frac{4z+3}{z^2-2z+3}$$

b) $G(z) = [1 \;\; 0]\begin{bmatrix} z+2 & 1 \\ -1 & z \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \dfrac{[1 \;\; 0]\begin{bmatrix} z & -1 \\ 1 & z+2 \end{bmatrix}}{z^2+2z+1}\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \dfrac{1}{z^2+2z+1}$

$$\frac{G}{1-GD} = \frac{\dfrac{1}{z^2+2z+1}}{1 - \dfrac{(4z+3)\cdot 1}{(z^2+2z+1)(z^2-2z+3)}} = \frac{z^2-2z+3}{z^2 \cdot z^2}$$

$$\begin{array}{rrrrrr} & 1 & 2 & 1 & & \\ 3 & -2 & 1 & & & \\ & 3 & -2 & 1 & & \\ & & 3 & -2 & 1 & \\ \hline & & & 3 & -2 & 1 \\ & & & & 3 & -2 \quad 1 \end{array}$$

$(z^4 + 0z^3 + 0z^2 + 4z + 3) - (4z+3) = z^4$

c) $\alpha_c(z) = |zI - \phi + \Gamma K| = \begin{vmatrix} z+2 & 1 \\ -4 & z-2 \end{vmatrix} = z^2$; $\alpha_e(z) = |zI - \phi + CH| = \begin{vmatrix} z & 1 \\ 0 & z \end{vmatrix}$

(CONTROLE DEAD-BEAT)

$$F = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \qquad H = [1 \ 0]$$

ESTABILIDADE : $|sI - F| = 0 \longrightarrow \begin{vmatrix} s-1 & 1 \\ 0 & s-1 \end{vmatrix} = 0$ PÓLO DUPLO EM $s = +1$ (INSTÁVEL)

CONTROLABILIDADE : $\mathcal{C} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$ — SISTEMA CONTROLÁVEL

OBSERVABILIDADE : $\Theta = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$ — SISTEMA OBSERVÁVEL

~~QUESTÃO #4:~~ QUESTÃO #3:

a) CONSIDERE $z = Px$, COM $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\left(T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = P^{-1} \right)$

$$\phi_z = T^{-1}\phi T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \left[\begin{array}{c|c} 2 & 1 \\ \hline -1 & 0 \end{array}\right]$$

ϕ_{zaa}, ϕ_{zab}, ϕ_{zba}, ϕ_{zbb}

$$\Gamma_z = T^{-1}\Gamma = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \cancel{X}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{matrix} \leftarrow \Gamma_{za} \\ \leftarrow \Gamma_{zb} \end{matrix}$$

$H_z = HT = [0 \ 1]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = [1 \ 0]$ (H_z ADEQUADO AO CÁLCULO DE L).

$$|zI - \phi_{zbb} + L\phi_{zab}| = \alpha_e(z)$$

$$z + L = z - 0.1 \implies L = -0.1$$

b) $u(k) = \text{\#\#\#} - K\hat{x}(k) = -KT\hat{z}(k) = -1.5\,y(k) - 1.0\,\hat{z}_2(k)$ (I)

$$K_z = KT = [1.0 \ \ 1.5]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = [1.5 \ \ 1.0]$$

DO FORMULÁRIO, TEMOS :

$$\hat{z}_2(k+1) = \underbrace{(\phi_{zbb} - L\phi_{zab})}_{+0.1}\hat{z}_2(k) + \underbrace{(\phi_{zba} - L\phi_{zaa})}_{-0.8}\underbrace{z_a(k)}_{y(k)} + \underbrace{(\Gamma_{zb} - L\Gamma_{za})}_{1}u(k) + \underbrace{L z_a(k+1)}_{-0.1\,y(k+1)}$$

$$\hat{z}_2(k+1) = 0.1\hat{z}_2(k) - 0.8\,y(k) + u(k) - 0.1\,y(k+1) \qquad (\text{II})$$

SUBSTITUINDO A EQUAÇÃO (I) NA EQUAÇÃO (II):

$$\hat{z}_2(k+1) = 0.1\hat{z}_2(k) - 0.8\,y(k) - 1.5\,y(k) - \hat{z}_2(k) - 0.1\,y(k+1)$$

ENTÃO :

$$\boxed{\begin{aligned} \hat{z}_2(k+1) &= -0.9\,\hat{z}_2(k) - 2.3\,y(k) - 0.1\,y(k+1) \\ u(k) &= -1.5\,y(k) - \hat{z}_2(k) \end{aligned}}$$

c) $z_2(z+0.9) = Y(-2.3 - 0.1z)$

$$z_2 = \frac{(-2.3 - 0.1z)Y}{z+0.9} \qquad (\text{III})$$

SUBSTITUINDO A EQUAÇÃO (III) NA EQUAÇÃO (I):

$$U = -1.5\,Y + \frac{(0.1z + 2.3)Y}{z+0.9} = \frac{(-1.4z + 0.95)Y}{z+0.9} \longrightarrow \boxed{D(z) = \frac{-1.4z + 0.95}{z+0.9}}$$

d) $$G(z) = [1 \ 0]\begin{bmatrix} z-2 & -1 \\ 1 & z \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = [1 \ 0]\frac{\begin{bmatrix} z & 1 \\ -1 & z-2 \end{bmatrix}}{z^2 - 2z + 1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{z^2 - 2z + 1}$$

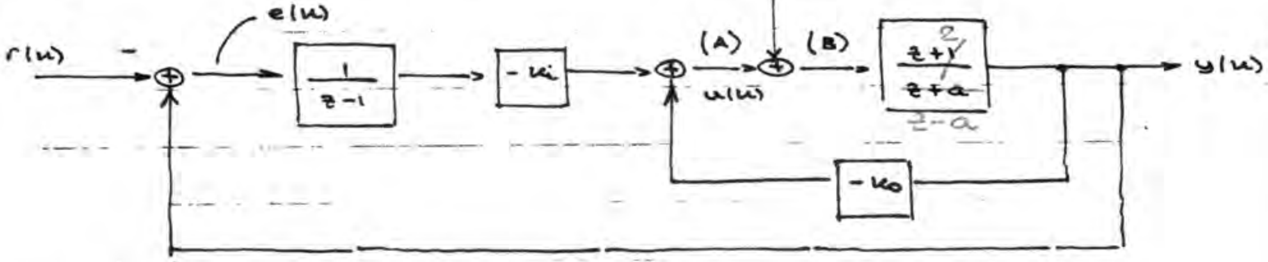
$$\frac{Y(z)}{R(z)} = \frac{\dfrac{1}{z^2 - 2z + 1}}{1 - \dfrac{1}{z^2 - 2z + 1}\cdot\dfrac{(-1.4z + 0.95)}{z+0.9}} = \frac{z+0.9}{(z^2 - 2z + 1)(z+0.9) + 1.4z - 0.95}$$

$$\frac{Y(z)}{R(z)} = \frac{z+0.9}{z^3 + (0.9-2)z^2 + (-1.8+1+1.4)z + 0.9 - 0.95} \longrightarrow \boxed{\frac{Y(z)}{R(z)} = \frac{z+0.9}{z^3 - 1.1z^2 + 0.6z - 0.05}}$$

OBS.: $\left.\begin{array}{l}\alpha_c(z) = (z-0.5-0.5j)(z-0.5+0.5j)\\ \alpha_e(z) = z-0.1\end{array}\right\}$ ESTA VERIFICAÇÃO NÃO É NECESSÁRIO.

QUESTÃO #5:

a)



b) $U(z) = \frac{-k_i}{z-1}\left(Y(z) - R(z)\right) - k_o Y(z)$ (A) (ASSUMINDO $w(k)=0$)

$$Y(z)\frac{\overset{z-a}{\cancel{z+a}}}{\underset{z+2}{\cancel{z+1}}} = \frac{-k_i}{z-1}Y(z) + \frac{k_i}{z-1}R(z) - k_o Y(z)$$

$$Y(z)\left(\frac{\overset{z-a}{\cancel{z+a}}}{\underset{z+2}{\cancel{z+1}}} + \frac{k_i}{z-1} + k_o\right) = \frac{k_i}{z-1}R(z)$$

$$\frac{Y(z)}{R(z)} = \frac{\frac{k_i}{z-1}}{\frac{\overset{(z-a)}{\cancel{z+a}}}{\underset{(z+2)}{\cancel{z+1}}} + \frac{k_i}{z-1} + k_o} = \frac{\overset{k_i(z+2)}{\cancel{k_i(z+1)}}}{\underset{(z-a)(z-1)+k_i(z+2)+k_o(z^2+z-2)}{\cancel{(z+a)(z-1)+k_i(z+1)+k_o(z^2-1)}}}$$

$$\frac{Y(z)}{R(z)} = \frac{\cancel{k_i(z+1)}\ k_i(z+2)}{(k_o+1)z^2 + \underset{(-a-1+k_i+k_o)z}{\cancel{(a-1+k_i)z}} + \cancel{k_i-k_o-a} + (a+2k_i-2k_o)}$$

c) $\alpha_c(z) = z^2 \longrightarrow \left\{\begin{array}{l} k_i + k_o = a+1\\ 2k_i - 2k_o = -a\\ 4k_i = a+2 \to k_i = \frac{a+2}{4}\\ -4k_o = -3a-2 \to k_o = \frac{3a+2}{4}\end{array}\right.$

SE $k_o+1 \neq 0$ (OU SEJA, SE $\overset{a\neq -2}{\cancel{a\neq 1}}$)

~~· $a-1+k_i=0 \longrightarrow k_i = 1-a$~~

~~· $k_i-k_o-a=0 \longrightarrow 1-a-k_o-a=0 \longrightarrow k_o = 1-2a$~~

d) $W(z) - k_o Y(z) - \frac{k_i Y(z)}{z-1} = Y(z)\frac{\overset{z-a}{\cancel{z+a}}}{\cancel{z+1}\ z+2}$ (B) (ASSUMINDO $r(k)=0$)

$$W(z)\underset{}{\overset{(z-1)(z+2)}{\cancel{(z^2-1)}}} = Y(z)\left(\overset{z-a}{\cancel{(z+a)}}(z-1) + k_o\overset{(z^2+z-2)}{\cancel{(z^2-1)}} + \overset{k_i(z+2)}{\cancel{k_i(z+1)}}\right)$$

$$W(z)\overset{(z-1)(z+2)}{\cancel{(z^2-1)}} = Y(z)\left(z^2(k_o+1) + \underset{(-a-1+k_i+k_o)}{\cancel{z(k_i+a-1)}} + \overset{a+2k_i-2k_o}{\cancel{k_i-k_o-a}}\right)$$

$$\frac{Y(z)}{W(z)} = \frac{\cancel{z^2-1}\ (z-1)(z+2)}{z^2(k_o+1) + \underset{(-a-1+k_i+k_o)z}{\cancel{z(k_i+a-1)}} + \cancel{k_i-k_o-a} + (a+2k_i-2k_o)}$$

SE $\cancel{k_i = 1-a}$ E $\cancel{k_o = 1-2a}$: $k_i = \frac{a+2}{4}$ $k_o = \frac{3a+2}{4}$

$$\frac{Y(z)}{W(z)} = \cancel{\frac{z^2-1}{z^2}\cdot\frac{1}{2-2a}} = \frac{(z-1)(z+2)}{z^2}\cdot\frac{4}{(3a+6)}$$

SE $W(z) = \frac{z}{z-1} \Longrightarrow Y(z) = \frac{4}{3a+6}\cdot\frac{z+2}{z} = \frac{4}{3a+6} + \frac{8z^{-1}}{3a+6}$

$y(0) = \frac{4}{3a+6}$; $y(1) = \frac{8}{3a+6}$; $y(k) = 0$, SE $k \geq 2$.

QUESTÃO #4:

a) $\hat{x}_b(z) = \left((zI - \phi_d)^{-1} \Gamma_{d1} + (zI - \phi_d)^{-1} z \Gamma_{d2} \right) Y(z)$

$$\phi_d = \begin{bmatrix} 464.5 & -404.6 \\ -297.7 & 260.5 \end{bmatrix} \qquad \Gamma_{d1} = \begin{bmatrix} -24.9 \\ 15.6 \end{bmatrix} \qquad \Gamma_{d2} = \begin{bmatrix} -49 \\ 32 \end{bmatrix}$$

$$U(z) = H_d \hat{x}_b(z) - 0.4 Y(z)$$

$$\frac{U(z)}{Y(z)} = D(z) = H_d (zI - \phi_d)^{-1} \Gamma_{d1} + z \left(H_d (zI - \phi_d)^{-1} \Gamma_{d2} \right) - 0.4$$

$$D(z) = \frac{-0.4z^2 + 169.13z - 145.899}{z^2 - 725z + 552.83} + \frac{(-241.5z^2 + 208.39z)}{z^2 - 725z + 552.83}$$

$$D(z) = \frac{-241.9z^2 + 377.52z - 145.879}{z^2 - 725z + 552.83} = \frac{n_d(z)}{d_d(z)}$$

b) $G(z) = \dfrac{3z^2 - 4.8z + 1.91}{z^3 - 2.4z^2 + 1.91z - 0.504} = \dfrac{n_g(z)}{d_g(z)}$

$$\frac{Y(z)}{R(z)} = \frac{G(z)}{1 - G(z)D(z)} = \frac{n_g(z) d_d(z)}{d_g(z) d_d(z) - n_g(z) n_d(z)} = \frac{n(z)}{d(z)}$$

$$n(z) = 3z^4 - 2179.8z^3 + 5140.40z^2 - 4038.334z + 1055.9053$$

$$d(z) = z^5 - 727.4z^4 + 2294.74z^3 - 2712.046z^2 + 1421.3053z - 278.62632$$
$$+ \; 725.70z^4 - 2293.68z^3 + 2711.756z^2 - 1421.2728z + 278.62507$$

$$d(z) = z^5 - 1.7z^4 + 1.06z^3 - 0.29z^2 + 0.0325z - 0.00125$$

c) $\alpha_c(z)\alpha_e(z) = (z^3 - 1.5z^2 + 0.75z - 0.125)(z^2 - 0.2z + 0.01)$

$$\alpha_c(z)\alpha_e(z) = z^5 - 1.7z^4 + 1.06z^3 - 0.29z^2 + 0.0325z - 0.00125$$

d) OS PÓLOS DO SISTEMA EM MALHA FECHADA SÃO: 0.5 (TRIPLO) E 0.1 (DUPLO).