Universidade Federal do Rio de Janeiro
Escola Politécnica

2008/2

DATA: 14/11/2008

GRAUS: 1, 2, 3, 4, 5

Aluno: GABARITO DA PROVA PARCIAL #2

Disciplina: CONTROLE LINEAR II-A

Turma: EEL760

Professor: JOSÉ GABRIEL

QUESTÃO #1:

a) $U(s) = \dfrac{1}{s(s+1)} = \dfrac{1}{s} - \dfrac{1}{s+1} \longrightarrow u(t) = (1 - e^{-t})u(t)$

b) $U(z) = \dfrac{z(1-e^{-T})}{(z-1)(z-e^{-T})}$; $\dfrac{U(z)}{z} = \dfrac{1}{z-1} + \dfrac{(-1)}{z-e^{-T}}$

$u(k) = (1 - e^{-Tk})u(k)$

MÉTODOS: MAPEAMENTO PÓLOS E ZEROS ; EQUIVALÊNCIA NA RESPOSTA AO DEGRAU ; DISCRETIZAÇÃO NO ESPAÇO DE ESTADOS.

c) $U(z) = \left(\dfrac{T}{T+2}\right) \cdot \dfrac{z(z+1)}{(z-1)(z-a)}$; $a = -\left(\dfrac{T-2}{T+2}\right)$

$\dfrac{U(z)}{z} = \dfrac{T}{T+2}\left(\dfrac{z+1}{(z-1)(z-a)}\right) = \dfrac{T}{T+2}\left(\dfrac{\frac{2}{1-a}}{z-1} + \dfrac{\frac{a+1}{a-1}}{z-a}\right)$

$\dfrac{2}{1-a} = \dfrac{T+2}{T}$ E $\dfrac{a+1}{a-1} = \dfrac{-2}{T}$

$\dfrac{U(z)}{z} = \dfrac{1}{z-1} + \dfrac{\left(\frac{-2}{T+2}\right)}{z-a} \longrightarrow u(k) = \left(1 - \dfrac{2}{T+2}\left(\dfrac{-(T-2)}{(T+2)}\right)^k\right)u(k)$

MÉTODO: BILINEAR

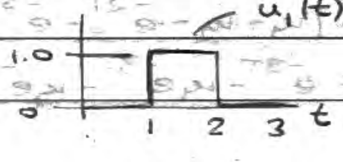
d) $U(z) = \dfrac{zT}{(z-1)(z+T-1)}$; $\dfrac{U(z)}{z} = T\left(\dfrac{1/T}{z-1} + \dfrac{(-1/T)}{z+T-1}\right)$

$\dfrac{U(z)}{z} = \dfrac{+1}{z-1} + \dfrac{(-1)}{z+T-1} \longrightarrow u(k) = (1 - (1-T)^k)u(k)$

MÉTODO: FORWARD EULER

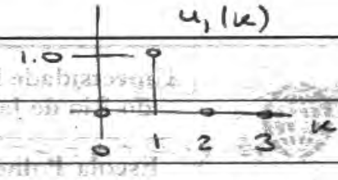
QUESTÃO #2:

$y_1(3) = 1 - e^{-2} + (-1 + e^{-1}) = e^{-1} - e^{-2}$

a)


$y_1(0) = 0$; $y_1(1) = 0$; $y_1(2) = 1 - e^{-1}$; $y_1(3) = $ ✓

b) $\dfrac{1-e^{-1}}{z-e^{-1}} = \dfrac{Y_1(z)}{U_1(z)} \longrightarrow y_1(k+1) = (1-e^{-1})\,u_1(k) + e^{-1} y_1(k)$



$y_1(0) = (1-e^{-1})\times 0 + e^{-1}\times 0 = 0$

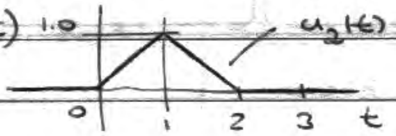
$y_1(1) = (1-e^{-1})\times 0 + e^{-1}\times 0 = 0$

$y_1(2) = (1-e^{-1})\times 1 + e^{-1}\times 0 = (1-e^{-1})$

$y_1(3) = (1-e^{-1})\times 0 + e^{-1}(1-e^{-1}) = (e^{-1}-e^{-2})$

→ NOTE QUE OS VALORES SÃO IGUAIS DOS CALCULADOS NO ITEM (a).

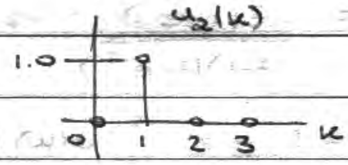
c)



$y_2(0) = 0\ ;\quad y_2(1) = e^{-1}\ ;\quad y_2(2) = 1 + e^{-2} - 2(e^{-1}) = 1 - 2e^{-1} + e^{-2}$

$y_2(3) = (2+e^{-3}) - 2(1+e^{-2}) + e^{-1} = e^{-1} - 2e^{-2} + e^{-3}$

d) $\dfrac{e^{-1}z + 1 - 2e^{-1}}{z-e^{-1}} = \dfrac{Y_2(z)}{U_2(z)}$



$y_2(k+1) = e^{-1} u_2(k+1) + (1-2e^{-1})\,u_2(k) + e^{-1} y_2(k)$

$y_2(0) = e^{-1}\times 0 + (1-2e^{-1})\times 0 + e^{-1}\times 0 = 0$

$y_2(1) = e^{-1}\times 1 + (1-2e^{-1})\times 0 + e^{-1}\times 0 = e^{-1}$

$y_2(2) = e^{-1}\times 0 + (1-2e^{-1})\times 1 + e^{-1}\times e^{-1} = 1 - 2e^{-1} + e^{-2}$

$y_2(3) = e^{-1}\times 0 + (1-2e^{-1})\times 0 + e^{-1}\times(1-2e^{-1}+e^{-2}) = e^{-1} - 2e^{-2} + e^{-3}$

NOTE QUE OS VALORES CALCULADOS SÃO IGUAIS DOS CALCULADOS NO ITEM (c).

EXPLICAÇÃO DE $G_2(z)$:

$Y_2(s) = \dfrac{G(s)}{s^2} = \dfrac{1}{s^2(s+1)} = \dfrac{1}{s^2} + \dfrac{(-1)}{s} + \dfrac{1}{s+1} \longrightarrow y_2(t) = (t-1+e^{-t})\,u(t)$

$y_2(k) = (k-1+e^{-k})\,u(k)$ (NOTE QUE $T=1$)

$Y_2(z) = \dfrac{z}{(z-1)^2} - \dfrac{z}{z-1} + \dfrac{z}{z-e^{-1}}$

→ MÉTODO: EQUIVALÊNCIA NA RESPOSTA À RAMPA (FIRST-ORDER HOLD).

$G_2(z) = \left(\dfrac{(z-1)^2}{z}\right)\left(\dfrac{z}{(z-1)^2} - \dfrac{z}{z-1} + \dfrac{z}{z-e^{-1}}\right)$

$G_2(z) = 1 - (z-1) + \dfrac{(z-1)^2}{z-e^{-1}} = \dfrac{z - e^{-1} - z^2 + e^{-1}z + z - e^{-1} + z^2 - 2z + 1}{z-e^{-1}} = \dfrac{e^{-1}z + 1 - 2e^{-1}}{z-e^{-1}}$ //

QUESTÃO #3:

a) $|zI - \phi + \Gamma K| = \begin{vmatrix} z-e^{-2T}+k_1 & k_2 \\ k_1 & z-e^{-3T}+k_2 \end{vmatrix} = z^2 + (k_1+k_2-e^{-2T}-e^{-3T})z + e^{-5T} - k_1e^{-3T} - k_2e^{-2T}$

$$\alpha_c(z) = (z-0.5)(z-0.2) = z^2 - 0.7z + 0.1$$

$$\begin{cases} k_1 + k_2 = -0.7 + e^{-2T} + e^{-3T} \\ e^{-3T}k_1 + e^{-2T}k_2 = e^{-5T} - 0.1 \end{cases} \Rightarrow \begin{cases} k_1 + k_2 = 0.86 \\ 0.74k_1 + 0.82k_2 = 0.51 \end{cases}$$

$$0.08k_2 = -0.13$$

$$K = [2.49 \quad -1.63] \longleftarrow k_2 = -1.63 \Longrightarrow k_1 = 2.49$$

b) $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$\phi_z = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e^{-2T} & 0 \\ 0 & e^{-3T} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^{-2T} & e^{-3T} \\ 0 & e^{-3T} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \left[\begin{array}{c|c} e^{-2T} & e^{-3T} - e^{-2T} \\ \hline 0 & e^{-3T} \end{array}\right]$$

($e^{-2T} \to 0.82$; $e^{-3T} - e^{-2T} \to -0.08$; $e^{-3T} \to 0.74$)

$$\left(\Gamma_z = T^{-1}\Gamma = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad H_z = HT = [1 \quad 0] \right.$$

$$\left. K_z = KT = [2.49 \mid -4.12] \right)$$

$$\rightarrow |zI - \phi_{bb} + L\phi_{ab}| = z - e^{-3T} + L(e^{-3T} - e^{-2T})$$

$$\alpha_e(z) = z \ (\text{DEAD-BEAT}) \Longrightarrow L = e^{-3T}/(e^{-3T} - e^{-2T}) = -9.25$$

c) $\hat{x}_2(k+1) = (\phi_{bb} - L\phi_{ab})\hat{x}_2(k) + (\phi_{ba} - L\phi_{aa})y(k) + (\Gamma_b - L\Gamma_a)u(k) + Ly(k+1)$

$$\phi_{bb} - L\phi_{ab} = 0.74 + 9.25 \times (-0.08) = 0$$

$$\phi_{ba} - L\phi_{aa} = 9.25 \times 0.82 = 7.59$$

$$\Gamma_b - L\Gamma_a = 1 + 9.25 \times 2 = 19.5$$

$$(\Gamma_b - L\Gamma_a)(-k_{z1}y(k) - k_{z2}x_2(k)) = 19.5(-2.49y(k) + 4.12x_2(k))$$

$$= -48.56y(k) + 80.34x_2(k)$$

ENTÃO: $zX_2 = 7.59Y - 48.56Y + 80.34X_2 - 9.25zY$

$$X_2 = \frac{-9.25z - 40.97}{z - 80.34}$$

E PORTANTO: $U = -2.49Y + 4.12\left(\dfrac{-9.25Y - 40.97}{z - 80.34}\right)Y$

$$\boxed{\frac{U(z)}{Y(z)} = \frac{-40.6z + 31.25}{z - 80.34}} \longleftarrow (D(z))$$

d) $G(z) = \dfrac{1}{z-0.82} + \dfrac{1}{z-0.74} = \dfrac{2z - 1.56}{z^2 - 1.56z + 0.6068}$

$$\frac{Y(z)}{R(z)} = \frac{G(z)}{1 - D(z)G(z)} = \frac{\dfrac{2z - 1.56}{z^2 - 1.56z + 0.6068}}{1 + \dfrac{(40.6z - 31.25)(2z - 1.56)}{(z - 80.34)(z^2 - 1.56z + 0.6068)}}$$

1	−1.56	0.6068		40.6	−31.25	
−80.34	1		1	−1.56	2	81.2
−80.34	1		−81.9	−1.56	2	−125.83
	−80.34	1	125.93		−1.56	48.75
		−80.34	−48.75			

$$-81.9 + 81.2 = -0.7 \qquad 125.93 - 125.83 = 0.1$$

PORTANTO: $\frac{Y(z)}{R(z)} = \frac{2(z-0.78)(z-80.34)}{z^3 - 0.7z^2 + 0.1z} \longleftarrow (z^2 - 0.7z + 0.1)z$ ($\alpha_c(z)$, $\alpha_e(z)$)

OBS.: NOTE O ZERO DE $\gamma(z)$ EM 80.34!

QUESTÃO #4: O ESTIMADOR DE ESTADOS É ATUALIZADO.

a) $\alpha_c(z) = \begin{vmatrix} z - 0.585 + 0.078 \times 4.656 & 0.468 + 0.078 \times 33.068 \\ -0.078 + 0.004 \times 4.656 & z - 0.975 + 0.004 \times 33.068 \end{vmatrix}$

$\alpha_c(z) = \begin{vmatrix} z - 0.22 & 3.05 \\ -0.06 & z - 0.84 \end{vmatrix} = z^2 - 1.06z + 0.37$

b) $\phi - LH\phi = \begin{bmatrix} 0.585 & -0.468 \\ 0.078 & 0.975 \end{bmatrix} - \begin{bmatrix} 4.927 & 4.927 \\ -4.150 & -4.150 \end{bmatrix} \begin{bmatrix} 0.585 & -0.468 \\ 0.078 & 0.975 \end{bmatrix}$

$= \begin{bmatrix} -2.68 & -2.97 \\ 2.83 & 3.08 \end{bmatrix}$

$\alpha_e(z) = \begin{vmatrix} z + 2.68 & 2.97 \\ -2.83 & z - 3.08 \end{vmatrix} = z^2 - 0.4z + 0.15$

c) $\gamma(z) = \begin{vmatrix} z + 1.164 & -7.813 \\ -1.226 & z + 8.305 \end{vmatrix} = z^2 + 9.47z + 0.09$

d) RAÍZES DE $\alpha_c(z)$: $0.53 \pm 0.3j = 0.61 e^{0.52j}$ (com $0.61 = \sqrt{0.53^2 + 0.3^2}$ e $0.52 = \operatorname{atan}(0.3/0.53)$)

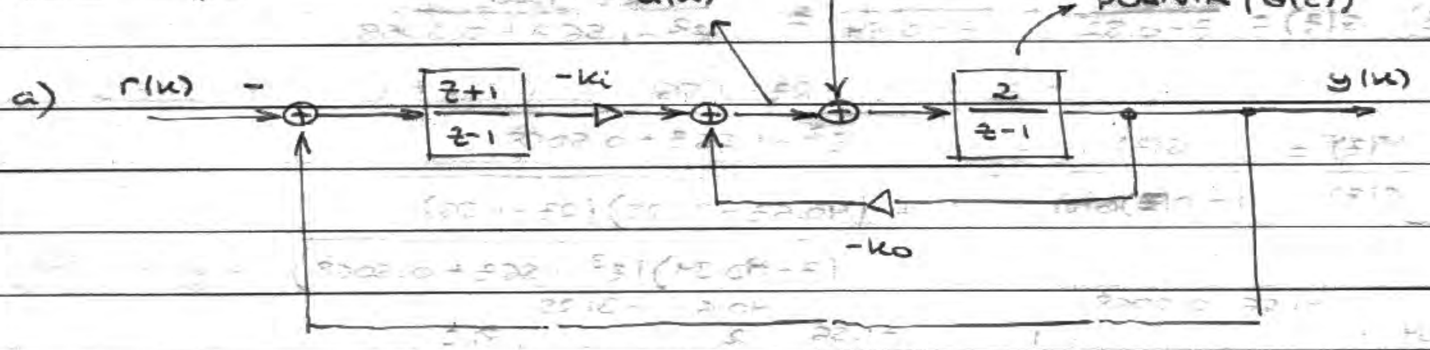
$0.61 e^{0.52j} = e^{\sigma T} e^{j\omega_d T}$

$T\sigma = \ln 0.61 \longrightarrow \sigma = \frac{-0.49}{T} = -4.9 \longrightarrow t_s \simeq 0.94 \text{ seg}$

$\omega_d T = 0.52 \longrightarrow \omega_d = 5.2 \longrightarrow t_p \simeq 0.60 \text{ seg}$

QUESTÃO #5:

a)



b) $Y(z)\left(\frac{z-1}{2}\right) = W(z) - k_i\left(\frac{z+1}{z-1}\right)\left(Y(z) - R(z)\right) - k_o Y(z)$

$$(z-1)^2 Y = 2(z-1)W - 2k_i z Y + 2k_i e z - 2k_i Y + 2k_i e - k_o z Y + k_o Y$$

$$(z^2 - 2z + 1 + 2k_i z + 2k_i + 2k_o z - k_o)\, Y = 2(z-1)W + 2k_i(z+1)e$$

$$Y(z) = \frac{(2z-2)W(z) + (2z+2)k_i e(z)}{z^2 + (2k_i + 2k_o - 2)z + 2k_i - k_o + 1}$$

$$\alpha_c(z) = (z-0.5)(z-0.2) = z^2 - 0.7z + 0.1$$

ENTÃO: $2k_i + 2k_o = 1.3$

$$\begin{array}{r} 2k_i - k_o = -0.9 \\ (-)\ \overline{\qquad 3k_o = 2.2} \end{array} \longrightarrow k_o = 0.73$$

$$k_i = \frac{-0.9 + 0.73}{2} \longrightarrow k_i = -0.09$$

c) $$\frac{Y(z)}{e(z)} = \frac{-0.18(z+1)}{z^2 - 0.7z + 0.1}$$

d) $$\frac{Y(z)}{W(z)} = \frac{2(z-1)}{z^2 - 0.7z + 0.1}$$

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