(9)

Universidade Federal do Rio de Janeiro
Escola Politécnica

DATA: 20/06/08

GRAUS:	
1	2.0
2	2.0
3	2.0
4	2.0
5	1.9
	9.9

Aluno: FLAVIA CORREIA TOVO — P2

Disciplina: CONTROLE II

Turma:

Professor: JOSÉ GABRIEL

1) $D(s) = \frac{s+1}{s^2}$

a) Mapeamento de pólos e zeros.

Zero: $s = -1 \xrightarrow{e^{sT}} z = e^{-T}$

PÓLO: DUPLO EM $s=0 \xrightarrow{e^{sT}} z = 1$ (DUPLO)

$$D(z) = \frac{z - e^{-T}}{(z-1)^2} \qquad 0.5$$

b) Integração numérica bilinear:

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

$$D(z) = \frac{\frac{2}{T}\left(\frac{z-1}{z+1}\right)+1}{\frac{4}{T^2}\left(\frac{z-1}{z+1}\right)^2} = \frac{\frac{2}{T}(z-1)(z+1)+(z+1)^2}{\frac{4}{T^2}(z-1)^2} = \frac{T^2}{4}\,\frac{\frac{2}{T}(z^2-1)+z^2+2z+1}{z^2-2z+1}$$

$$D(z) = \frac{z^2\left(\frac{T}{2}+\frac{T^2}{4}\right) + z\left(\frac{T^2}{2}\right) + \frac{T^2}{4} - \frac{T}{2}}{z^2-2z+1} \qquad 0.5$$

c) Equivalência na resposta ao degrau.

$$\frac{D(s)}{s} = \frac{s+1}{s^3} = Y(s) \qquad A + Bs + Cs^2 = s+1$$

$A=1 \quad C=0$

$$Y(s) = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} \qquad B=1$$

$$Y(s) = \frac{1}{s^3} + \frac{1}{s^2} \longrightarrow y(t) = \left(t + \frac{t^2}{2}\right)u(t) \longrightarrow y_A(k) = \left(KT + \frac{K^2T^2}{2}\right)u(K)$$

$$Y_A(z) = \frac{zT}{(z-1)^2} + \frac{T^2}{2}\frac{z(z+1)}{(z-1)^3} \;;\; D(z) = Y_A(z)\cdot\frac{(z-1)}{z} = \frac{T}{(z-1)} + \frac{T^2}{2}\frac{(z+1)}{(z-1)^2}$$

$$D(z) = \frac{z\left(T+\frac{T^2}{2}\right)+\frac{T^2}{2}-T}{z^2-2z+1} \qquad 0.5$$

d) Discretização no espaço de estados.

$$F=\begin{bmatrix}0&0\\1&0\end{bmatrix};\quad G=\begin{bmatrix}1\\0\end{bmatrix};\quad H=\begin{bmatrix}1&1\end{bmatrix}$$

$$\Phi = \mathcal{L}^{-1}\left((sI-F)^{-1}\right)\Big|_{t=T} = \mathcal{L}^{-1}\left(\begin{bmatrix}s&0\\-1&s\end{bmatrix}^{-1}\right)\Bigg|_{t=T} = \mathcal{L}^{-1}\left(\begin{bmatrix}\frac{1}{s}&0\\\frac{1}{s^2}&\frac{1}{s}\end{bmatrix}\right)\Bigg|_{t=T}$$

$$\Phi = \begin{bmatrix}u(T)&0\\T\,u(T)&u(T)\end{bmatrix} = \begin{bmatrix}1&0\\T&1\end{bmatrix}$$

$$\Gamma = \int_0^T e^{Ft}G\,dt = \int_0^T\begin{bmatrix}u(t)\\t\,u(t)\end{bmatrix}dt = \begin{bmatrix}T\\\frac{T^2}{2}\end{bmatrix}$$

2.0/2.0

$$D(z) = H(zI-\Phi)^{-1}\Gamma = \frac{\begin{bmatrix}1&1\end{bmatrix}\begin{bmatrix}z-1&0\\T&z-1\end{bmatrix}\begin{bmatrix}T\\\frac{T^2}{2}\end{bmatrix}}{(z-1)^2} \qquad 0.5$$

$$D(z) = \frac{\begin{bmatrix}z-1+T & z-1\end{bmatrix}\begin{bmatrix}T\\\frac{T^2}{2}\end{bmatrix}}{(z-1)^2} = \frac{z\left(T+\frac{T^2}{2}\right)+\frac{T^2}{2}-T}{z^2-2z+1}$$

Igual a ⓒ como esperado.

② $$x(k+1) = \begin{bmatrix}\Phi_{aa}=0{,}5 & \Phi_{ab}=1\\ \Phi_{ba}=0{,}5 & \Phi_{bb}=0\end{bmatrix}x(k) + \begin{bmatrix}\Gamma_a=1\\\Gamma_b=0\end{bmatrix}u(k);\quad y=\begin{bmatrix}H_a=1 & H_b=0\end{bmatrix}x(k)$$

a) $\alpha_e(z) = z-0{,}1$

$|z-0+L\cdot 1| = z-0{,}1$

$z+L = z-0{,}1 \Rightarrow L=-0{,}1$ 0.5

b) $K = \begin{bmatrix}K_a=1 & K_b=1\end{bmatrix}$

$\hat{x}_b(k+1) = (0-(-0{,}1)\,1)\,\hat{x}_b(k) + (0{,}5-(-0{,}1)\,0{,}5)\,x_a(k) + (0-(-0{,}1)\,1)\,u(k) - 0{,}1\,x_a(k+1)$

$x_a(k) = y(k)$

$u(k) = -K_a\,y(k) - K_b\,\hat{x}_b(k)$

$\hat{x}_b(k+1) = (0{,}1 - 1\cdot 0{,}1)\,\hat{x}_b(k) + (0{,}55 - 1\cdot 0{,}1)\,y(k) - 0{,}1\,y(k+1)$

$\hat{x}_b(k+1) = 0{,}45\,y(k) - 0{,}1\,y(k+1)$ 0.5

c) $D(z) = ?$

$$z\hat{X}_1(z) = (0{,}45 - 0{,}1z)\,Y(z)$$

$$U(z) = -Y(z) - \frac{(0{,}45 - 0{,}1z)\,Y(z)}{z}$$

$$D(z) = \frac{U(z)}{Y(z)} = \frac{(-z - 0{,}45 + 0{,}1z)}{z}$$

$$D(z) = \frac{-0{,}9z - 0{,}45}{z}$$ 0.5/

2.0/2.0

d) $$G(z) = \frac{z}{z^2 - 0{,}5z - 0{,}5}$$

$$\frac{Y(z)}{R(z)} = \frac{G(z)}{1 - G(z)D(z)} = \frac{z^2}{z^3 - 0{,}5z^2 - 0{,}5z + 0{,}9z^2 + 0{,}45z}$$

$$\frac{Y(z)}{R(z)} = \frac{z^2}{(z^2 + 0{,}4z - 0{,}05)z} = \frac{z^2}{\underbrace{(z - 0{,}1)}_{\alpha_e(z)}\underbrace{(z + 0{,}5)z}_{\alpha_c(z)}}$$ ✓ 0.5/

$$\alpha_c(z) = |zI - \Phi + \Gamma K| = \begin{vmatrix} z - 0{,}5 + 1 & 0 \\ -0{,}5 & z \end{vmatrix} = z(z + 0{,}5)$$

$$\frac{Y(z)}{R(z)} = \frac{z}{(z - 0{,}1)(z + 0{,}5)}$$ → o z de ✓ $\alpha_c(z)$ "some" pelo cancelamento entre pólo e zero.

③ $$G(z) = \frac{1}{z^3 - 1}$$ (FCO) a) $$\hat{x}(k+1) = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}}_{\Phi - LH - \Gamma K} \hat{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} y(k)$$ 0.5/

$\hookrightarrow L$

$$u(k+1) = \underbrace{\begin{bmatrix} -1 & 0 & 0 \end{bmatrix}}_{-K} \hat{x}(k+1)$$

b) $$D(z) = -\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z & -1 & 0 \\ 0 & z & -1 \\ +1 & 0 & z \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

só preciso do termo (1, 3) da inversa: (3, 1) da cofatora

$$D(z) = \frac{+1\,(-1)^4\,(1)\,1}{z^3 + 1} = \frac{-1}{z^3 + 1}$$ 0.5/

$$\frac{Y(z)}{R(z)} = \frac{n_g \cdot d_d}{d_g \cdot d_d - n_g \cdot n_d}$$ ✓

$$(a+b)(a-b) = a^2 - b^2$$

c) $$\frac{Y(z)}{R(z)} = \frac{z^3 + 1}{\underbrace{z^6 - 1}_{(z^3+1)(z^3-1)} + 1} = \frac{z^3 + 1}{z^6}$$ ✓ 0.5/

d) $\dfrac{Y(z)}{R(z)} = \dfrac{z^3+1}{z^6} = \alpha_c(z) \cdot \alpha_e(z)$

$H = [1 \quad 0 \quad 0]$; $LH = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; $\Gamma = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; $\Gamma K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ +1 & 0 & 0 \end{bmatrix}$

2.0 / 2.0

$\phi = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\alpha_c(z) = \begin{vmatrix} z & -1 & 0 \\ 0 & z & -1 \\ 0 & 0 & z \end{vmatrix} = z^3$ 0.5/

$\alpha_e(z) = \begin{vmatrix} z & -1 & 0 \\ 0 & z & -1 \\ 0 & 0 & z \end{vmatrix} = z^3$

6 pólos em $z=0$

3 zeros em ~~z = -1~~

(b) $X(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$

$y = [1 \quad 0] \, x(k)$

a) $L_a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $LH = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$; $LH\phi = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\alpha_e(z) = |zI - \phi + LH\phi|$

$\alpha_e(z) = \begin{vmatrix} z-1+1 & -1+1 \\ +1 & z-1+1 \end{vmatrix} = z^2$ dead-beat 0.5/

b) $K = [1 \quad 2]$; $\Gamma K = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$

$\alpha_c(z) = |zI - \phi + \Gamma K| = \begin{vmatrix} z-1 & -1 \\ 1 & z-1+2 \end{vmatrix}$

$\alpha_c(z) = (z-1)(z+1) + 1 = z^2 - 1 + 1$

$\alpha_c(z) = z^2$ como esperavamos 0.5/

dead-beat

c) $D(z) = ?$

$D(z) = -K(zI - \phi + LH\phi + \Gamma K - LH\Gamma K)^{-1} \cdot L \cdot z$; $LH\Gamma K = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= -[1 \quad 2] \begin{bmatrix} z & 0 \\ 2 & z+2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot z = \dfrac{(z-2+2z)\,z}{z(z+2)}$

$D(z) = \dfrac{-3z+2}{z+2}$ 0.5/

Universidade Federal
do Rio de Janeiro

Escola Politécnica

DATA 20 / 06 / 08

GRAUS:

1	
2	
3	
4	
5	

Aluno: FLAVIA CORREIA TOVO

Disciplina: CONTROLE II

Turma: –

Professor: JOSÉ GABRIEL.

CONTINUAÇÃO

4

me interessa esse termo, que na inversa muda de sinal.

d) $G(z) = [1 \ \ 0]\begin{bmatrix} z-1 & -1 \\ 0 & z-1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \dfrac{1}{(z-1)^2} = \dfrac{1}{z^2-2z+1}$

2.0 / 2.0

$\dfrac{G(z)}{1-D(z)G(z)} = \dfrac{z+2}{z^3 - \cancel{2z^2} + \cancel{2z^2} + \cancel{z} - \cancel{4z} + \cancel{2} + \cancel{3z} - \cancel{2}}$

0.5 /

$\dfrac{G(z)}{1-D(z)G(z)} = \dfrac{z+2}{z^3}$

sofreu um cancelamento entre pólo e zero. seria $\frac{z^2+2z}{z^4}$, mas isso não afeta o $\lim_{z\to 1}$

$\lim_{z\to 1} \dfrac{G(z)}{1-D(z)G(z)} = 3$

5

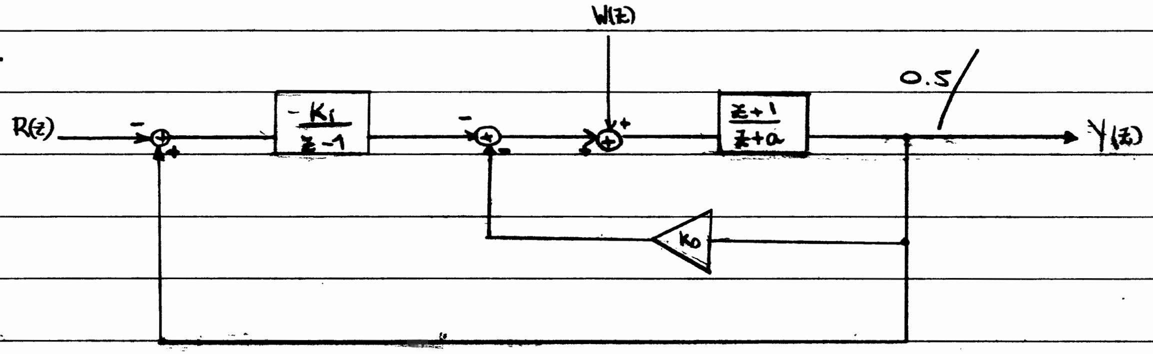
$E(z) = Y(z) - R(z)$

$U(z) = \dfrac{-K_i}{z-1}E(z) - K_o Y(z)$

$Y(z) = \dfrac{z+1}{z+a}\left(U(z) + W(z)\right)$

a)

0.5 /



b) $\dfrac{Y(z)}{R(z)}$

$$Y(z) = \frac{z+1}{z+a} U(z) \longleftarrow U(z) = -\frac{k_i}{z-1} E(z) - k_o Y(z)$$

$$\left(1 + k_o \frac{z+1}{z+a}\right) Y(z) = -\frac{z+1}{z+a} \frac{k_i}{z-1} E(z) \longleftarrow E(z) = Y(z) - R(z)$$

$$\left(1 + k_o \frac{z+1}{z+a} + \frac{k_i}{z-1}\frac{z+1}{z+a}\right) Y(z) = \frac{z+1}{z+a}\frac{k_i}{z-1} R(z)$$

$$\frac{Y(z)}{R(z)} = \frac{k_i(z+1)}{(z-1)(z+a) + k_o(z+1)(z-1) + k_i(z+1)}$$

$$\frac{Y(z)}{R(z)} = \frac{k_i(z+1)}{z^2(1+k_o) + z(a-1+k_i) - a - k_o + k_i}$$ 0.5/

c) $a - 1 + k_i = 0 \Rightarrow k_i = 1 - a$

$-a + k_i - k_o = 0 \Rightarrow k_o = 1 - 2a$ 0.5/

$(a \neq 1)$

$$\frac{Y(z)}{R(z)} = \frac{(1-a)(z+1)}{(2-2a)z^2} = \frac{z+1}{2z^2}$$

d) $$U(z) = \frac{a-1}{z-1} Y(z) + (2a-1) Y(z)$$

1.9/2.0

$$Y(z) = \frac{z+1}{z+a}\frac{a-1}{z-1} Y(z) + \frac{z+1}{z+a}(2a-1) Y(z) + \frac{z+1}{z+a} W(z)$$

$$\frac{Y(z)}{W(z)} = \frac{\frac{z+1}{z+a}}{1 + \frac{z+1}{z+a}\frac{1-a}{z-1} + \frac{z+1}{z+a}(1-2a)}$$

$$\frac{Y(z)}{W(z)} = \frac{(z+1)(z-1)}{(z+a)(z-1) + (z+1)(1-a) + (z+1)(z-1)(1-2a)}$$ 0.4/

$$\frac{Y(z)}{W(z)} = \frac{z^2-1}{2z^2 + z(a-1+1-a) - a + 1 - a - 1 + 2a}$$

$$\frac{Y(z)}{W(z)} = \frac{z^2-1}{2z^2}$$

Teste: $\lim_{z\to 1} \dfrac{Y(z)}{W(z)} = 0$ (como esperado)

$(2-2a)$ ✗ -0.1 (OK. VERIFICAR CÁLCULOS.)