Universidade Federal do Rio de Janeiro

Escola Politécnica

DATA: 26/11/2007

GRAUS:

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Aluno: GABARITO DA PROVA PARCIAL

Disciplina: CONTROLE LINEAR II

Turma: 2007/2

Professor: JOSÉ GABRIEL

QUESTÃO #1

a)
$$D(z) = \frac{\frac{z-1}{T}}{\left(\frac{z-1}{T}\right)^2 + 1} = \frac{T(z-1)}{z^2 - 2z + (T^2+1)} \overset{T=0.1}{=} \frac{0.1(z-1)}{z^2 - 2z + 1.01} = \frac{Y(z)}{U(z)}$$

$$y(k) = 0.1\left(u(k-1) - u(k-2)\right) + 2y(k-1) - 1.01\,y(k-2)$$

RESPOSTA DO DEGRAU (CONDIÇÕES INICIAIS $y(-1) = y(-2) = 0$):

$k=0$: $y(0) = 0$

$k=1$: $y(1) = 0.1 \times 1 + 2 \times 0 = 0.1$

$k=2$: $y(2) = 0.1 \times 0 + 2 \times 0.1 - 1.01 \times 0 = 0.2$

$k=3$: $y(3) = 0.1 \times 0 + 2 \times 0.2 - 1.01 \times 0.1 = 0.299$

b)
$$D(z) = \frac{\left(\frac{z-1}{Tz}\right)}{\left(\frac{z-1}{Tz}\right)^2 + 1} = \frac{T(z^2 - z)}{z^2(T^2+1) - 2z + 1} = \frac{0.1(z^2 - z)}{1.01z^2 - 2z + 1} = \frac{Y(z)}{U(z)}$$

$$y(k) = \frac{1}{1.01}\left(0.1\left(u(k) - u(k-1)\right) + 2y(k-1) - y(k-2)\right) \qquad \begin{pmatrix} y(-1) = 0 \\ y(-2) = 0 \end{pmatrix}$$

$k=0$: $y(0) = 0.1/1.01 = 0.099$

$k=1$: $y(1) = 2 \times 0.099/1.01 = 0.196$

$k=2$: $y(2) = (2 \times 0.196 - 0.099)/1.01 = 0.290$

$k=3$: $y(3) = (2 \times 0.290 - 0.196)/1.01 = 0.380$

c)
$$F_{cc} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \; ; \; G_{cc} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \; ; \; H_{cc} = [1 \;\; 0]$$

$$(sI - F_{cc})^{-1} = \begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix}^{-1} = \frac{\begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}}{s^2+1} \; ; \; \mathcal{L}^{-1}\left((sI - F_{cc})^{-1}\right) = \begin{bmatrix} \cos(t)u(t) & -\sin(t)u(t) \\ \sin(t)u(t) & \cos(t)u(t) \end{bmatrix}$$

$$\phi = e^{F_{cc}T} = \begin{bmatrix} \cos(T) & -\sin(T) \\ \sin(T) & \cos(T) \end{bmatrix} \xrightarrow{T=0.1} \phi = \begin{bmatrix} 0.995 & -0.100 \\ 0.100 & 0.995 \end{bmatrix}$$

$$\Gamma = \int_0^T \begin{bmatrix} \cos(t) \\ \sin(t) \end{bmatrix} dt = \begin{bmatrix} \sin(t)\Big|_0^T \\ -\cos(t)\Big|_0^T \end{bmatrix} = \begin{bmatrix} \sin(T) \\ 1 - \cos(T) \end{bmatrix} \xrightarrow{T=0.1} \Gamma = \begin{bmatrix} 0.100 \\ 0.005 \end{bmatrix}$$

$$H = H_{cc} = [1 \;\; 0]$$

$k=0$: ($x(0)=0$; CONDIÇÕES INICIAIS NULOS) $\longrightarrow$ $y(0)=0$

$k=1$: $x(1)=\begin{bmatrix}0.100\\0.005\end{bmatrix} \longrightarrow y(1)=0.1$

$k=2$: $x(2)=\begin{bmatrix}0.995 & -0.100\\0.100 & 0.995\end{bmatrix}\begin{bmatrix}0.100\\0.005\end{bmatrix}+\begin{bmatrix}0.100\\0.005\end{bmatrix}=\begin{bmatrix}0.199\\0.020\end{bmatrix} \longrightarrow y(2)=0.199$

$k=3$: $x(3)=\begin{bmatrix}0.995 & -0.100\\0.100 & 0.995\end{bmatrix}\begin{bmatrix}0.199\\0.020\end{bmatrix}+\begin{bmatrix}0.100\\0.005\end{bmatrix}=\begin{bmatrix}0.296\\0.045\end{bmatrix} \longrightarrow y(3)=0.296$

d) $y(0)=\sin(0)=0$; $y(1)=\sin(0.1)=0.1$; $y(2)=\sin(0.2)=0.199$

$y(3)=\sin(0.3)=0.296$ (RESULTADOS IGUAIS AOS DO ITEM (c)).

QUESTÃO #2

a) $\Theta_1=\begin{bmatrix}0.781 & -0.483\\0.469 & -0.287\end{bmatrix}$; $\det\Theta_1=0.0021$

$\Theta_2=\begin{bmatrix}1.045 & -0.509\\0.489 & -0.233\end{bmatrix}$; $\det\Theta_2=0.0054$

b) $Y(z)=H(zI-\phi)^{-1}x(0)z$

SISTEMA #1: $Y(z)=[0.781 \;\; -0.483]\dfrac{\begin{bmatrix}z & -0.368\\1 & z-1.219\end{bmatrix}}{z^2-1.219z+0.368}\begin{bmatrix}1\\0\end{bmatrix}z=\dfrac{0.781z^2-0.483z}{(z-0.55)(z-0.669)}$

$$\frac{1}{2}\left(1.219\pm\sqrt{1.219^2-4\times0.368}\right)=\begin{cases}\to 0.550\\ \to 0.669\end{cases}$$

$$\frac{Y(z)}{z}=\frac{0.781z-0.483}{(z-0.55)(z-0.669)}=\frac{A}{z-0.55}+\frac{B}{z-0.669}$$

$A=\left.\dfrac{0.781z-0.483}{z-0.669}\right|_{z=0.55}=0.449$; $B=\left.\dfrac{0.781z-0.483}{z-0.55}\right|_{z=0.669}=0.332$

$y(k)=\left(0.449(0.55)^k+0.332(0.669)^k\right)u(k)$

SISTEMA #2: $Y(z)=[1.045 \;\; -0.509]\dfrac{\begin{bmatrix}z & -0.223\\1 & z-0.955\end{bmatrix}}{z^2-0.955z+0.223}=\dfrac{1.045z^2-0.509z}{(z-0.407)(z-0.548)}$

$$\frac{1}{2}\left(0.955\pm\sqrt{0.955^2-4\times0.223}\right)=\begin{cases}\to 0.407\\ \to 0.548\end{cases}$$

$$\frac{Y(z)}{z}=\frac{1.045z-0.509}{(z-0.407)(z-0.548)}=\frac{A}{z-0.407}+\frac{B}{z-0.548}$$

$A=\left.\dfrac{1.045z-0.509}{z-0.548}\right|_{z=0.407}=0.594$; $B=\left.\dfrac{1.045z-0.509}{z-0.407}\right|_{z=0.548}=0.452$

$y(k)=\left(0.594(0.407)^k+0.45\cancel{2}_{2}(0.548)^k\right)u(k)$

c) $y(k) = \left(2 - e^{-2Tk} - e^{-3Tk}\right) u(k)$ EQUIV. RESPOSTA DO DEGRAU

$$Y(z) = \frac{2z}{z-1} - \frac{z}{z-e^{-2T}} - \frac{z}{z-e^{-3T}} \qquad \left(G(z) = \frac{z-1}{z} Y(z) \right)$$

$$G(z) = 2 - \frac{z-1}{z-e^{-2T}} - \frac{z-1}{z-e^{-3T}} = \frac{\left(-2e^{-2T} - 2e^{-3T} + e^{-3T} + 1 + e^{-2T} + 1\right) z + \left(2e^{-5T} - e^{-3T} - e^{-2T}\right)}{z^2 - (e^{-2T} + e^{-3T}) z + e^{-5T}}$$

$$G(z) = \frac{\left(2 - e^{-2T} - e^{-3T}\right) z + 2e^{-5T} - e^{-3T} - e^{-2T}}{z^2 - (e^{-2T} + e^{-3T}) z + e^{-5T}}$$

d)

	$T = 0.2$	$T = 0.3$
$2 - e^{-2T} - e^{-3T}$	0.781	1.045
$2e^{-5T} - e^{-3T} - e^{-2T}$	−0.483	−0.509
$-(e^{-2T} + e^{-3T})$	−1.219	−0.955
e^{-5T}	0.368	0.223
	(SISTEMA #1)	(SISTEMA #2)

SISTEMA #1: $\phi_{co} = \begin{bmatrix} 1.219 & 1 \\ -0.368 & 0 \end{bmatrix}$ $\quad \Gamma_{co} = \begin{bmatrix} 0.781 \\ -0.483 \end{bmatrix}$ $\quad H_{co} = [1 \ 0]$

SISTEMA #2: $\phi_{co} = \begin{bmatrix} 0.955 & 1 \\ -0.223 & 0 \end{bmatrix}$ $\quad \Gamma_{co} = \begin{bmatrix} 1.045 \\ -0.509 \end{bmatrix}$ $\quad H_{co} = [1 \ 0]$

QUESTÃO #3:

a) $z_1, z_2 = 0.7 \pm 0.7j$

$e^{sT} = e^{x+jy} = z \implies e^{x}(\cos y + j \sin y) = 0.7\sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + j \sin\left(\frac{\pi}{4}\right)\right)$

$x = \log(0.7\sqrt{2}) = -0.0101$

$y = \pi/4 = 0.7854 \longrightarrow T = 0.1 \longrightarrow s = -0.101 \pm 7.854j$

b) $\phi_{cc} = \begin{bmatrix} 1.4 & -0.98 \\ 1 & 0 \end{bmatrix}$ $\quad \Gamma_{cc} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad H_{cc} = [1 \ 1]$

c) $e^{sT} = e^{-0.8574}\left(\cos(0.7854) \pm j \sin(0.7854)\right) = 0.3 \pm 0.3j$

$\alpha_c(z) = (z - 0.3 + 0.3j)(z - 0.3 - 0.3j) = z^2 - 0.6z + 0.18 \longrightarrow K = [0.8 \ -0.8]$

$z^2 - 1.4z + 0.98 \longleftarrow$ DENOMINADOR DE $G(z)$

d) $\phi_{cc} - \Gamma_{cc} K = \begin{bmatrix} 0.6 & -0.18 \\ 1 & 0 \end{bmatrix}$

$$\frac{Y(z)}{R(z)} = \frac{[1 \ 1] \begin{bmatrix} z & -0.18 \\ 1 & z-0.6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{z^2 - 0.6z + 0.18} = \frac{z+1}{z^2 - 0.6z + 0.18} \quad (\text{SE } \bar{N} = 1)$$

GANHO DC: $\frac{2}{1 - 0.6 + 0.18} = \frac{2}{0.58} = 3.4483 \longrightarrow \bar{N} = 0.29$

QUESTÃO #4:

a) $D(z) = -\overbrace{[0.25 \;\; 0.75]}^{K} \overbrace{\begin{bmatrix} z & 0 \\ 0.75 & z+0.75 \end{bmatrix}^{-1}}^{(zI-\phi+LH\phi-\Gamma K+LH\Gamma K)} \overbrace{\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}}^{L} z = -[0.25 \;\; 0.75] \dfrac{\begin{bmatrix} z+0.75 & 0 \\ -0.75 & z \end{bmatrix}}{z^2+0.75z} \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} z$

$D(z) = -[0.25 \;\; 0.75] \dfrac{\begin{bmatrix} z+0.75 \\ 0.5z-0.75 \end{bmatrix}}{z^2+0.75z} z = \dfrac{(-0.625z+0.375)\cancel{z}}{z^2+0.75\cancel{z}} = \dfrac{-0.625z+0.375}{z+0.75}$

b) $G(z) = [1 \;\; 0] \begin{bmatrix} z-1 & -2 \\ 0 & z-1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = [1 \;\; 0] \dfrac{\begin{bmatrix} z-1 & 2 \\ 0 & z-1 \end{bmatrix}}{z^2-2z+1} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \dfrac{2z+2}{z^2-2z+1}$

$\dfrac{Y(z)}{R(z)} = \dfrac{\dfrac{2z+2}{z^2-2z+1}}{1 + \dfrac{2z+2}{z^2-2z+1} \cdot \dfrac{(0.625z-0.375)}{z+0.75}} = \dfrac{2z^2 + 3.5z + 1.5}{z^3}$.

	2	2			2	2	
0.75	1		2	−0.375	0.625		~~1.25~~
	0.75	1	3.5		−0.375	0.625	~~0.5~~
		0.75	1.5			−0.375	~~−0.75~~

	1	−2	1		
0.75	1				1
	0.75	1			~~−1.25~~
		0.75	1		~~−0.5~~
			0.75		~~+0.75~~

c) $\phi - LH\phi = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} [1 \;\; 2] = \begin{bmatrix} 0 & 0 \\ -0.5 & 0 \end{bmatrix}$

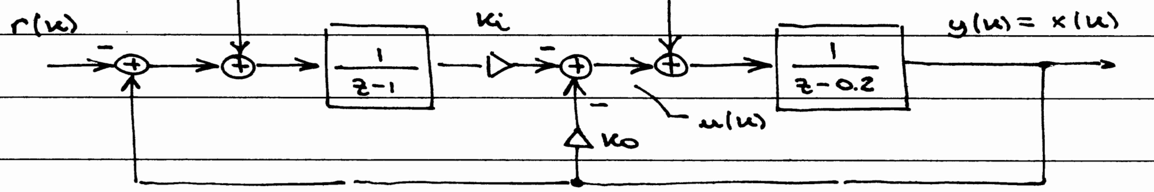
$|zI - \phi + LH\phi| = \begin{vmatrix} z & 0 \\ 0.5 & z \end{vmatrix} = z^2 \longrightarrow$ L FOI CALCULADO PARA QUE $\alpha_e(z) = z^2$ (DEAD-BEAT).

d) $\dfrac{Y(z)}{R(z)} = \dfrac{(2z+2)\overbrace{(z+0.75)z}^{\gamma(z)}}{\underbrace{z^2}_{\alpha_c(z)} \cdot \underbrace{z^2}_{\alpha_e(z)}}$

GANHO DC: $2+3.5+1.5 = 7.0$

QUESTÃO #5:

a)



$y(k) = [0 \;\; 1] \begin{bmatrix} x_i \\ x \end{bmatrix}$

b) $\begin{bmatrix} x_i(k+1) \\ x(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0.2 \end{bmatrix} \begin{bmatrix} x_i(k) \\ x(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [-k_i \;\; -k_o] \begin{bmatrix} x_i(k) \\ x(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_1(k) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} r(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w_2(k)$

$\phi_c = \begin{bmatrix} 1 & 1 \\ -k_i & 0.2-k_o \end{bmatrix}$; $|zI - \phi_c| = \begin{vmatrix} z-1 & -1 \\ k_i & z+k_o-0.2 \end{vmatrix} = z^2 + (k_o - 1.2)z + k_i - k_o + 0.2$

$k_o = 1.2$ E $k_i = 1.0$ PARA QUE $\alpha_c(z) = z^2 \implies K = [1.0 \;\; 1.2]$

c) $\dfrac{Y(z)}{W_1(z)} = [0 \;\; 1] \begin{bmatrix} z-1 & -1 \\ 1 & z+1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [0 \;\; 1] \dfrac{\begin{bmatrix} z+1 & 1 \\ -1 & z-1 \end{bmatrix}}{z^2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \implies \dfrac{Y(z)}{W_1(z)} = \dfrac{z-1}{z^2}$

d) $\dfrac{Y(z)}{W_2(z)} = [0 \;\; 1] \dfrac{\begin{bmatrix} z+1 & 1 \\ -1 & z-1 \end{bmatrix}}{z^2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \dfrac{-1}{z^2} \implies \dfrac{Y(z)}{W_2(z)} = \dfrac{-1}{z^2}$