Universidade Federal do Rio de Janeiro

Escola Politécnica

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GRAUS: 1, 2, 3, 4, 5

Aluno: GABARITO DA PROVA PARCIAL #2

Disciplina: CONTROLE II

Turma: 2007/1

Professor: GABRIEL

QUESTÃO #1:

a) $D(s) = \dfrac{s+1}{s^2+6s+8}$

ZERO: $-1 \xrightarrow{e^{sT}} e^{-0.05} = 0.95$

PÓLOS: $-2 \xrightarrow{e^{sT}} e^{-0.1} = 0.9$

$-4 \xrightarrow{e^{sT}} e^{-0.2} = 0.82$

$$D(z) = \frac{K(z-0.95)}{(z-0.9)(z-0.82)} = \frac{0.045(z-0.95)}{(z-0.9)(z-0.82)}$$

$$\lim_{s\to 0} D(s) = \frac{1}{8} \qquad \lim_{z\to 1} D(z) = \frac{0.05\,K}{0.1 \times 0.18} = 2.78K \implies K = 0.045$$

b) $$Y(z) = \frac{z}{z-1} D(z) = 0.045 \frac{z(z-0.95)}{(z-1)(z-0.9)(z-0.82)} \qquad \frac{-0.05}{(-0.1)(0.08)} = 6.25$$

$$\frac{1}{0.045}\frac{Y(z)}{z} = \frac{(z-0.95)}{(z-1)(z-0.9)(z-0.82)} = \frac{2.78}{z-1} + \frac{6.25}{z-0.9} + \frac{(-9.03)}{z-0.82}$$

$$y(k) = \left(0.13 + 0.28(0.9)^k - 0.41(0.82)^k\right)u(k) \qquad \frac{-0.13}{(-0.18)(-0.08)} = -9.03$$

c) $$e^{Ft} = \mathcal{L}^{-1}\left((sI-F)^{-1}\right) = \mathcal{L}^{-1}\left[\begin{pmatrix} s+6 & 8 \\ -1 & s \end{pmatrix}^{-1}\right] = \mathcal{L}^{-1}\left[\frac{\begin{bmatrix} s & -8 \\ 1 & s+6 \end{bmatrix}}{s^2+6s+8}\right]$$

$$\frac{1}{(s+2)(s+4)} = \frac{1/2}{s+2} + \frac{(-1/2)}{s+4}$$

$$\frac{s}{(s+2)(s+4)} = \frac{-1}{s+2} + \frac{2}{s+4} \implies e^{Ft} = \begin{bmatrix} -e^{-2t}+2e^{-4t} & -4e^{-2t}+4e^{-4t} \\ \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t} & 2e^{-2t} - e^{-4t} \end{bmatrix}$$

$$\phi = e^{FT} = \begin{bmatrix} -e^{-0.1}+2e^{-0.2} & -4e^{-0.1}+4e^{-0.2} \\ 0.5e^{-0.1} - 0.5e^{-0.2} & 2e^{-0.1} - e^{-0.2} \end{bmatrix} = \begin{bmatrix} 0.74 & -0.32 \\ 0.04 & 0.98 \end{bmatrix}$$

$$\int_0^T \left(-e^{-2t} + 2e^{-4t}\right) dt = \left.\frac{e^{-2t}}{2} - \frac{e^{-4t}}{2}\right|_0^T = \frac{e^{-2T} - e^{-4T}}{2} = 0.04$$

$$\int_0^T \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t}\right) dt = \left.\frac{-e^{-2t}}{4} + \frac{e^{-4t}}{8}\right|_0^T = \frac{-e^{-2T}}{4} + \frac{1}{4} + \frac{e^{-4T}}{8} - \frac{1}{8} = 0.0025$$

$$\Gamma = \begin{bmatrix} 0.04 \\ 0.0025 \end{bmatrix} \qquad H = [1 \quad 1]$$

$$D(z) = [1 \quad 1] \begin{bmatrix} z-0.74 & 0.32 \\ -0.04 & z-0.98 \end{bmatrix}^{-1} \begin{bmatrix} 0.04 \\ 0.0025 \end{bmatrix} = \frac{[1 \quad 1] \begin{bmatrix} z-0.98 & -0.32 \\ 0.04 & z-0.74 \end{bmatrix} \begin{bmatrix} 0.04 \\ 0.0025 \end{bmatrix}}{z^2 - 1.72z + \cancel{0.74}}$$

0.738

$$D(z) = \frac{[z-0.94 \quad z-1.06] \begin{bmatrix} 0.04 \\ 0.0025 \end{bmatrix}}{z^2 - 1.72z + 0.738}$$

$$D(z) = \frac{0.043(z-0.95)}{(z-0.9)(z-0.82)}$$

d) A RESPOSTA AO DEGRAU É APROXIMADAMENTE IGUAL À DO ITEM (a), COM UM ERRO DE $\left(\frac{45-43}{43}\right) \times 100\%$.

QUESTÃO #2:

a) $$\frac{Y(s)}{R(s)} = \frac{\frac{1}{s}}{1 + \frac{1}{s} \cdot \frac{2}{s+3}} = \frac{s+3}{s^2+3s+2}$$

$$R(s) = \frac{1}{s} \longrightarrow Y(s) = \frac{s+3}{s(s+1)(s+2)} = \frac{3/2}{s} - \frac{2}{s+1} + \frac{1/2}{s+2}$$

$$y(t) = \left(\frac{3}{2} - 2e^{-t} + \frac{1}{2}e^{-2t}\right) u(t)$$

$$y(k) = \left(\frac{3}{2} - 2e^{-TK} + \frac{1}{2}e^{-2TK}\right) u(k)$$

b) $$\frac{Y(z)}{R(z)} = \frac{\frac{0.2}{z-1}}{1 + \frac{0.2}{z-1} \cdot \frac{2(z+1)}{(13z-7)}} = \frac{0.2(13z-7)}{13z^2 - 20z + 7 + 0.4z + 0.4} = \frac{0.2(13z-7)}{13z^2 - 19.6z + 7.4}$$

$$\frac{Y(z)}{R(z)} = \frac{0.2(z-0.5385)}{z^2 - 1.5077z + 0.5692}$$ (COM D(z) BILINEAR)

c) $$\frac{Y(z)}{R(z)} = \frac{\frac{0.2}{z-1}}{1 + \frac{0.2}{z-1} \cdot \frac{0.3}{z-0.55}} = \frac{0.2(z-0.55)}{z^2 - 1.55z + 0.61}$$ (COM D(z) EQUIV. DEGRAU)

d) $$Y_A(s) = \frac{1}{s} G(s) = \frac{1}{s^2} \longrightarrow y_A(t) = t\,u(t) \longrightarrow y_A(k) = T.k\,u(k)$$

$$\longrightarrow Y_A(z) = \frac{Tz}{(z-1)^2} \longrightarrow G(z) = \frac{z-1}{z} Y_A(z) = \frac{T}{z-1} = \frac{0.2}{z-1}$$

$$\frac{Y(z)}{R(z)} = \frac{\frac{0.2}{z-1}}{1 + \frac{0.2}{z-1} \cdot \frac{(0.29z+0.01)}{z-0.55}} = \frac{0.2(z-0.55)}{z^2 - 1.492z + 0.552}$$ (COM D(z) EXATO)

QUESTÃO #3 :

a) RESPOSTA DADA NO QUADRO. $K_{cm} = [0.4 \quad -2.7 \quad 3.2]$

b) $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $T = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\phi_z = T^{-1}\phi_{cm}T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.7 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\phi_z = \begin{bmatrix} 0.7 & 0.8 & 0.9 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \left[\begin{array}{c|cc} 0.7 & 0.1 & 0.2 \\ \hline 0 & 0.8 & 0 \\ 0 & 0 & 0.9 \end{array}\right] = \begin{bmatrix} \phi_{aa} & \phi_{ab} \\ \phi_{ba} & \phi_{bb} \end{bmatrix}$$

$$\Gamma_z = T^{-1}\Gamma_{cm} = \begin{bmatrix} 1 & +1 & +1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ \hline 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \Gamma_a \\ \Gamma_b \end{bmatrix}$$

$$H_z = H_{cm}T = [1 \quad 1 \quad 1]\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1 \quad 0 \quad 0]$$

c) $\alpha_e(z) = (z-0.1)^2 = z^2 - 0.2z + 0.01$

$$\phi_{bb} - L\phi_{ab} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.9 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}[0.1 \quad 0.2] = \begin{bmatrix} 0.8 - 0.1\ell_1 & -0.2\ell_1 \\ -0.1\ell_2 & 0.9 - 0.2\ell_2 \end{bmatrix}$$

$$|zI - \phi_{bb} + L\phi_{ab}| = \begin{vmatrix} z + 0.1\ell_1 - 0.8 & 0.2\ell_1 \\ 0.1\ell_2 & z + 0.2\ell_2 - 0.9 \end{vmatrix} =$$

$$= z^2 + (0.1\ell_1 + 0.2\ell_2 - 1.7) - 0.09\ell_1 - 0.16\ell_2 + 0.72$$

$$\begin{cases} 0.1\ell_1 + 0.2\ell_2 = 1.5 \\ -0.09\ell_1 - 0.16\ell_2 = -0.71 \end{cases} \qquad \begin{cases} \ell_1 + 2\ell_2 = 15 \\ 9\ell_1 + 16\ell_2 = 71 \end{cases} \qquad L = \begin{bmatrix} -49 \\ 32 \end{bmatrix}$$

$\ell_1 = -49 \longrightarrow \ell_2 = 32$

d) $K = K_{cm}T = [0.4 \quad -3.1 \quad 2.8]$

$$\hat{x}_b(k+1) = (\phi_{bb} - L\phi_{ab})\hat{x}_b + (\phi_{ba} - L\phi_{aa})y(k) + (\Gamma_b - L\Gamma_a)u(k) + Ly(k+1)$$

$$\phi_{bb} - L\phi_{ab} = \begin{bmatrix} 5.7 & 9.8 \\ -3.2 & -5.5 \end{bmatrix} \qquad \phi_{ba} - L\phi_{aa} = \begin{bmatrix} 34.3 \\ -22.4 \end{bmatrix} \qquad \Gamma_b - L\Gamma_a = \begin{bmatrix} 148 \\ -95 \end{bmatrix}$$

$$u(k) = -K_a y(k) - K_b\hat{x}_b(k) = -0.4y(k) - [-3.1 \quad 2.8]\hat{x}_b(k)$$

$$u(k) = -0.4y(k) + [3.1 \quad -2.8]\hat{x}_b(k)$$

$$\phi_{ba} - L\phi_{aa} + (\Gamma_b - L\Gamma_a)(-0.4) = \begin{bmatrix} 34.3 \\ -22.4 \end{bmatrix} + \begin{bmatrix} -59.2 \\ 38 \end{bmatrix} = \begin{bmatrix} -24.9 \\ 15.6 \end{bmatrix}$$

$$\phi_{bb} - L\phi_{ab} + (\Gamma_b - L\Gamma_a)[3.1 \quad -2.8] = \begin{bmatrix} 5.7 & 9.8 \\ -3.2 & -5.5 \end{bmatrix} + \begin{bmatrix} 458.8 & -414.4 \\ -294.5 & 266 \end{bmatrix} = \begin{bmatrix} 464.5 & -404.6 \\ -297.7 & 260.5 \end{bmatrix}$$

$$\begin{cases} \hat{x}_b(k+1) = \begin{bmatrix} 464.5 & -404.6 \\ -297.7 & 260.5 \end{bmatrix}\hat{x}_b(k) + \begin{bmatrix} -24.9 \\ 15.6 \end{bmatrix}y(k) + \begin{bmatrix} -49 \\ 32 \end{bmatrix}y(k+1) \\ u(k) = [3.1 \quad -2.8]\hat{x}_b(k) - 0.4y(k) \end{cases}$$

QUESTÃO #4:

a) $\alpha_e(z) = (z-0.1)^2 = z^2 - 0.2z + 0.01$

$H\phi = [0.4 \quad 1]$

$$\phi - LH\phi = \begin{bmatrix} 0.4 & 1 \\ -0.04 & 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} [0.4 \quad 1] = \begin{bmatrix} 0.4 - 0.4\ell_1 & 1-\ell_1 \\ -0.04 - 0.4\ell_2 & -\ell_2 \end{bmatrix}$$

$$|zI - \phi + LH\phi| = \begin{vmatrix} z + 0.4\ell_1 - 0.4 & \ell_1 - 1 \\ 0.4\ell_2 + 0.04 & z + \ell_2 \end{vmatrix} =$$

$$= z^2 + (0.4\ell_1 + \ell_2 - 0.4)z - \cancel{0.4\ell_2} + \cancel{0.4\ell_2} - 0.04\ell_1 + 0.04$$

$0.4\ell_1 + \ell_2 = 0.2$ $\qquad\qquad$ $\ell_1 = 3/4 = 0.75$

ENTÃO: $L = \begin{bmatrix} 0.75 \\ -0.1 \end{bmatrix}$

b) $\phi - LH\phi = \begin{bmatrix} 0.1 & 0.25 \\ 0 & 0.1 \end{bmatrix}$

$$-\Gamma K + LH\Gamma K = \begin{bmatrix} 0 \\ -1 \end{bmatrix} [0.05 \quad 0.2] + \underbrace{\begin{bmatrix} 0.75 \\ -0.1 \end{bmatrix} [1 \quad 0] \begin{bmatrix} 0 \\ 1 \end{bmatrix} [0.05 \quad 0.2]}_{\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right)} = \begin{bmatrix} 0 & 0 \\ -0.05 & -0.2 \end{bmatrix}$$

$\hat{x}(k+1) = (\phi - LH\phi - \Gamma K + LH\Gamma K)\hat{x}(k) + Ly(k+1)$

$u(k) = -K\hat{x}(k)$

ENTÃO:
$$\begin{cases} \hat{x}(k+1) = \begin{bmatrix} 0.1 & 0.25 \\ -0.05 & -0.1 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 0.75 \\ -0.1 \end{bmatrix} y(k+1) \\ u(k) = -[0.05 \quad 0.2]\,\hat{x}(k) \end{cases}$$

c) k AMOSTRAR $y(k)$ (LER DO CONVERSOR A/D)

$$\hat{x}(k) = \check{x}(k) + \begin{bmatrix} 0.75 \\ -0.1 \end{bmatrix} y(k)$$

$u(k) = -[0.05 \quad 0.2]\,\hat{x}(k)$

APLICAR $u(k)$ (ENVIAR AO CONVERSOR D/A)

$$\check{x}(k+1) = \begin{bmatrix} 0.1 & 0.25 \\ -0.05 & -0.1 \end{bmatrix} \hat{x}(k)$$

d) $G(z) = \dfrac{1}{z^2 - 0.4z + 0.04}$

$$D(z) = -[0.05 \quad 0.2] \begin{bmatrix} z-0.1 & -0.25 \\ 0.05 & z+0.1 \end{bmatrix}^{-1} \begin{bmatrix} 0.75 \\ -0.1 \end{bmatrix} z =$$

$$D(z) = -\frac{[0.05 \quad 0.2] \begin{bmatrix} z+0.1 & 0.25 \\ -0.05 & z-0.1 \end{bmatrix} \begin{bmatrix} 0.75 \\ -0.1 \end{bmatrix} z}{z^2 + 0.0025}$$

$$D(z) = \frac{-z\,[\,0.05z-0.005 \quad 0.2z-0.0075\,]\begin{bmatrix}0.75\\-0.1\end{bmatrix}}{z^2+0.0025} = \frac{-z\left(0.0175z-0.003\right)}{z^2+0.0025}$$

$$D(z) = \frac{-0.0175z^2+0.003z}{z^2+0.0025}$$

$$\frac{Y(z)}{R(z)} = \frac{\dfrac{1}{z^2-0.4z+0.04}}{1+\dfrac{1}{z^2-0.4z+0.04}\cdot\dfrac{(0.0175z^2-0.003z)}{z^2+0.0025}}$$

$$\frac{Y(z)}{R(z)} = \frac{z^2+0.0025}{z^4-0.4z^3+0.0425z^2-0.001z+\cancel{0.0001}+0.0175z^2-0.003z}$$

0.0001

$$\frac{Y(z)}{R(z)} = \frac{z^2+0.0025}{z^4-0.4z^3+0.06z^2-0.004z+0.0001} = \frac{\overset{\gamma(z)}{z^2+0.0025}}{\underbrace{(z^2-0.2z+0.01)}_{\alpha_c(z)}\underbrace{(z^2-0.2z+0.01)}_{\alpha_e(z)}}$$

QUESTÃO #5:

a) $$\left(\left(\frac{-k_i z}{z-1}\right)(Y-R)-k_o(Y+MR)\right)\frac{1}{z-0.4} = Y$$

$$Y(z-0.4)(z-1) = -k_i z(Y-R)-k_o(z-1)(Y+MR)$$

$$Y\left(z^2-1.4z+0.4\right) = -k_i zY + k_i zR - k_o(z-1)Y - k_o(zM-M)R$$

$$Y\left(z^2+(k_i+k_o-1.4)z+0.4-k_o\right) = R\left(z(k_i-k_oM)+k_oM\right)$$

$$\frac{Y}{R} = \frac{(k_i-k_oM)\left(z+\dfrac{k_oM}{k_i-k_oM}\right)}{z^2+(k_i+k_o-1.4)z+0.4-k_o} \leftarrow (\text{EQ.}1)$$

DEAD·BEAT: $k_o = 0.4$ E $k_i = 1.0$ $\quad(\alpha_c(z) = z^2)$

b) $$\frac{k_oM}{k_i-k_oM} = -0.5 \implies 0.4M = -0.5+0.2M$$

$$0.2M = -0.5 \implies M = -2.5$$

c) SUBSTITUINDO k_i, k_o E M NO EQ.1: $$\frac{Y(z)}{R(z)} = \frac{2(z-0.5)}{z^2}$$

d) $$\left(-\left(\frac{k_i z}{z-1}\right)Y-k_oY+W\right)\frac{1}{z-0.4} = Y$$

$$-k_i zY-k_o(z-1)Y+W(z-1) = Y(z-0.4)(z-1)$$

$$Y\left(z^2-1.4z+0.4+k_i z+k_o z-k_o\right) = W(z-1)$$

SUBSTITUINDO k_i E k_o DO ITEM (a): $$\frac{Y(z)}{W(z)} = \frac{(z-1)}{z^2}$$