CONTROLE II — SEGUNDO SEMESTRE DE 2006

GABARITO DA PROVA PARCIAL # 2 — 01/12/2006 — MODELO A

QUESTÃO # 1:

a) $$D(z) = \frac{\frac{2}{T}\left(\frac{z-1}{z+1}\right)}{\frac{2}{T}\left(\frac{z-1}{z+1}\right)+1} = \frac{(z-1)}{(z-1)+\frac{T}{2}(z+1)} = \frac{z-1}{\left(\frac{T}{2}+1\right)z+\frac{T}{2}-1}$$

b) $$Y(s) = \frac{D(s)}{s} = \frac{1}{s+1} \longrightarrow y(t) = e^{-t}u(t) \longrightarrow y(k) = e^{-kT}u(k)$$

$$Y(z) = \frac{z}{z-e^{-T}} \Longrightarrow D(z) = \left(\frac{z-1}{z}\right)Y(z) = \frac{z-1}{z-e^{-T}}$$

c) BILINEAR: $$Y_B(z) = \frac{z}{z-1}D(z) = \frac{z}{1.5z-0.5} = 0.67\frac{z}{z-0.33}$$

$$y_B(k) = 0.67(0.33)^k u(k)$$

EQUIV. DEGRAU:

$$Y_E(z) = \frac{z}{z-1}D(z) = \frac{z}{z-0.37} \Rightarrow y_E(k) = (0.37)^k u(k)$$

NOTE QUE $y_E(k)$ É IGUAL À SEQÜÊNCIA DE AMOSTRAS DA RESPOSTA AO DEGRAU DE $D(s)$: É $e^{-t}u(t)\Big|_{t=kT}$. A APROXIMAÇÃO BILINEAR TEM ERRO MAIS ELEVADO.

d) BILINEAR: $$Y_B(z) = \frac{z}{1.05z-0.95} = 0.95\frac{z}{z-0.90} \longrightarrow y_B(k) = 0.95(0.9)^k u(k)$$

EQUIV. DEGRAU: $$Y_E(z) = \frac{z}{z-0.90} \Rightarrow y_E(k) = 0.9^k u(k)$$

O ERRO DA APROXIMAÇÃO BILINEAR DIMINUIU, MAS $y_E(k)$ CONTINUA TENDO ERRO IGUAL A ZERO NOS INSTANTES $t = kT$.

QUESTÃO # 2:

a) $$e^{Ft} = \mathcal{L}^{-1}\left[\begin{bmatrix} s+3 & 2 \\ -1 & s \end{bmatrix}^{-1}\right] = \mathcal{L}^{-1}\begin{bmatrix} \frac{s}{(s+1)(s+2)} & \frac{-2}{(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} & \frac{s+3}{(s+1)(s+2)} \end{bmatrix}$$

$$\frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2} \longrightarrow (e^{-t}-e^{-2t})u(t)$$

$$\frac{s}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2} \longrightarrow (-e^{-t}+2e^{-2t})u(t)$$

$$\phi = e^{FT} = \begin{bmatrix} -e^{-T}+2e^{-2T} & -2e^{-T}+2e^{-2T} \\ e^{-T}-e^{-2T} & 2e^{-T}-e^{-2T} \end{bmatrix} = \begin{bmatrix} 2x^2-x & 2x^2-2x \\ -x^2+x & -x^2+2x \end{bmatrix} \qquad (x = e^{-T})$$

b) $$\int_0^T \begin{pmatrix} -e^{-t}+2e^{-2t} \\ e^{-t}-e^{-2t} \end{pmatrix} dt = \begin{bmatrix} e^{-t}-e^{-2t}\Big|_0^T \\ -e^{-t}+\frac{1}{2}e^{-2t}\Big|_0^T \end{bmatrix} = \begin{bmatrix} e^{-T}-1+1-e^{-2T} \\ 1-e^{-T}+\frac{1}{2}e^{-2T}-\frac{1}{2} \end{bmatrix} = \begin{bmatrix} e^{-T}-e^{-2T} \\ \frac{1}{2}-e^{-T}+\frac{1}{2}e^{-2T} \end{bmatrix} = \Gamma$$

$$= \begin{bmatrix} -x^2+x \\ \frac{1}{2}x^2-x+\frac{1}{2} \end{bmatrix}$$

c) $H\phi = \begin{bmatrix} a(2x^2-x) + b(-x^2+x) & a(2x^2-2x) + b(-x^2+2x) \end{bmatrix}$

$H\phi = \begin{bmatrix} x^2(2a-b) + x(b-a) & x^2(2a-b) + x(2b-2a) \end{bmatrix}$

$$\mathcal{O}_d = \begin{bmatrix} H \\ H\phi \end{bmatrix} = \begin{bmatrix} a & b \\ x^2(2a-b) + x(b-a) & x^2(2a-b) + x(2b-2a) \end{bmatrix}$$

$HF = \begin{bmatrix} b-3a & -2a \end{bmatrix}$

$$\mathcal{O}_c = \begin{bmatrix} H \\ HF \end{bmatrix} = \begin{bmatrix} a & b \\ b-3a & -2a \end{bmatrix}$$

d) SE $\det \mathcal{O}_c \neq 0$, ENTÃO $-2a^2 - b^2 + 3ab \neq 0 \implies 2a^2 - 3ab + b^2 \neq 0$

$\det \mathcal{O}_d = x^2(2a^2 - ab) + x(2ab - 2a^2) + x^2(b^2 - 2ab) + x(ab - b^2)$

$\det \mathcal{O}_d = x^2(2a^2 + b^2 - 3ab) - x(2a^2 + b^2 - 3ab)$

SE $\det \mathcal{O}_c \neq 0$ (E PORTANTO $2a^2 - 3ab + b^2 \neq 0$), ENTÃO $\det \mathcal{O}_d \neq 0$ DESDE QUE $x^2 - x \neq 0$, OU SEJA, $e^{-T} \neq 0$ E $e^{-T} \neq 1$.

$(T < \infty)$ $(T \neq 0)$

QUESTÃO #3:

a) $|zI - \phi + LH| = \begin{vmatrix} z-1.8+\ell_1 & -1 \\ 0.81+\ell_2 & z \end{vmatrix} = z^2 + (\ell_1 - 1.8)z + \ell_2 + 0.81 = z^2$

$\ell_1 = 1.8$ $\ell_2 = -0.81$ L_p: $L = \begin{bmatrix} 1.8 \\ -0.81 \end{bmatrix}$

b) $|zI - \phi + LH\phi| = \begin{vmatrix} z-1.8+1.8\ell_1 & -1+\ell_1 \\ 0.81+1.8\ell_2 & z+\ell_2 \end{vmatrix}$

$= z^2 + (1.8\ell_1 + \ell_2 - 1.8)z \cancel{-1.8\ell_2} + \cancel{1.8\ell_2} + 0.81 - 0.81\ell_1 = z^2$

$\ell_1 = 1 \longrightarrow \ell_2 = 0$ $\ell_1 = 1$ L_a: $L = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

OPCIONALMENTE: $L_a = \phi^{-1} L_p = \begin{bmatrix} 1.80 & 1 \\ -0.81 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1.8 \\ -0.81 \end{bmatrix}$

$= \frac{1}{0.81} \begin{bmatrix} 0 & -1 \\ 0.81 & 1.80 \end{bmatrix} \begin{bmatrix} 1.8 \\ -0.81 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

c) $\phi - LH\phi = \begin{bmatrix} 1.8 & 1 \\ -0.81 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1.8 & 1 \\ -0.81 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -0.81 & 0 \end{bmatrix}$

$\Gamma - LH\Gamma = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{0} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$-(\Gamma - LH\Gamma)K = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0.4 & 0.4 \end{bmatrix} = \left(\begin{bmatrix} 0 & 0 \\ -0.4 & -0.4 \end{bmatrix} \right)$

$$\hat{x}(k+1) = \begin{bmatrix} 0 & 0 \\ -1.21 & -0.4 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} y(k+1)$$

$$u(k) = -[0.4 \quad 0.4]\, \hat{x}(k)$$

ENTÃO $D(z) = -0.4\,[1 \quad 1] \begin{bmatrix} z & 0 \\ 1.21 & z+0.4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} z$

$$D(z) = -0.4\,[1 \quad 1] \frac{\begin{bmatrix} z+0.4 & 0 \\ -1.21 & z \end{bmatrix}}{z^2+0.4z} \begin{bmatrix} 1 \\ 0 \end{bmatrix} z = -0.4\,[1 \quad 1] \frac{\begin{bmatrix} z+0.4 \\ -1.21 \end{bmatrix}}{z^2+0.4z} z$$

$$D(z) = \frac{-0.4(z-0.81)}{z+0.4}$$

$\gamma(z) = (z+0.4)\cancel{z}$

d) $$\frac{G(z)}{1-D(z)G(z)} = \frac{\dfrac{1}{z^2-1.8z+0.81}}{1 + \dfrac{0.4(z-0.81)}{(z+0.4)(z^2-1.8z+0.81)}} = \frac{z+0.4}{z^3-1.4z^2+0.49z}$$

$\alpha_c(z) \cdot \alpha_e(z)$

$(z^2-1.4z+0.49)\,\cancel{z^2}$

NOTE QUE $|zI - \phi + \Gamma K|$

$$= \begin{vmatrix} z-1.8 & -1 \\ 1.21 & z+0.4 \end{vmatrix} = z^2 - 1.4z + 0.49$$

	1	-1.8	0.81			
0.4	1					
	0.4	1				
		0.4	1			
			0.4	1		

1	-1.4	0.09	0.324
		0.4	-0.324
1	-1.4	0.49	0

$\longrightarrow z^3 - 1.4z^2 + 0.49z$

QUESTÃO #4:

$-1.25\, x_a(k) + 0.75\, \hat{x}_b(k)$

a) $$\hat{x}_b(k+1) = \underbrace{(\phi_{bb} - L\phi_{ab})}_{0.2} \hat{x}_b(k) + \underbrace{(\phi_{ba} - L\phi_{aa})}_{0.6} x_a(k) + \underbrace{(\Gamma_b - L\Gamma_a)}_{-1.2} u(k) + L x_a(k+1)$$

$$\hat{x}_b(k+1) = 0.2\hat{x}_b(k) + 0.6y(k) + 1.5y(k) - 0.9\hat{x}_b(k) + 0.2y(k+1)$$

$$\hat{x}_b(k+1) = -0.7\hat{x}_b(k) + 2.1y(k) + 0.2y(k+1)$$

b) INSTANTE $t = kT$: k

- AMOSTRAR $y(k)$
- $\hat{x}_b(k) = \breve{x}_b(k) + 0.2y(k)$
- $u(k) = -1.25\,y(k) + 0.75\,\hat{x}_b(k)$
- ENVIAR $u(k)$ P/ CONVERSOR D/A
- $\breve{x}_b(k+1) = -0.7\hat{x}_b(k) + 2.1y(k)$

⋮

INSTANTE $t = (k+1)T$: $k+1$

⋮

c) $\hat{x}_B(z+0.7) = Y(0.2z+2.1)$

$$\frac{\hat{x}_B(z)}{Y(z)} = \frac{0.2z+2.1}{z+0.7}$$

$$U(z) = -1.25\,Y(z) + 0.75\,\hat{x}_B(z) = \left(-1.25 + \frac{0.75(0.2z+2.1)}{z+0.7}\right) Y(z)$$

$$\frac{U(z)}{Y(z)} = \frac{-1.25z - 0.875 + 0.15z + 1.575}{z+0.7} = \frac{-1.1z+0.7}{z+0.7} \quad \therefore \quad D(z) = \frac{-1.1z+0.7}{z+0.7}$$

d) $$G(z) = [1\ \ 0]\begin{bmatrix} z-2 & 1 \\ -1 & z \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{[1\ \ 0]\begin{bmatrix} z & -1 \\ 1 & z-2 \end{bmatrix}}{z^2-2z+1}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{z+1}{z^2-2z+1}$$

$$\frac{G(z)}{1-G(z)D(z)} = \frac{\dfrac{z+1}{z^2-2z+1}}{1+\dfrac{(z+1)(1.1z-0.7)}{(z^2-2z+1)(z+0.7)}} = \frac{(z+1)(z+0.7)}{z^2(z-0.2)}$$

JUSTIFICANDO:

- $b(z) = (z+1)$ ✓
- $\gamma(z) = (z+0.7)$ ✓
- $\alpha_c(z) = z^2$ ✓

NOTE QUE $|zI - \phi + \Gamma K|$

$$= \begin{vmatrix} z-\frac{3}{4} & \frac{1}{4} \\ -\frac{9}{4} & z+\frac{3}{4} \end{vmatrix} = z^2$$ ✓

- $\alpha_e(z) = (z-0.2)$

$= |zI - \phi_{bb} + L\phi_{ab}|$

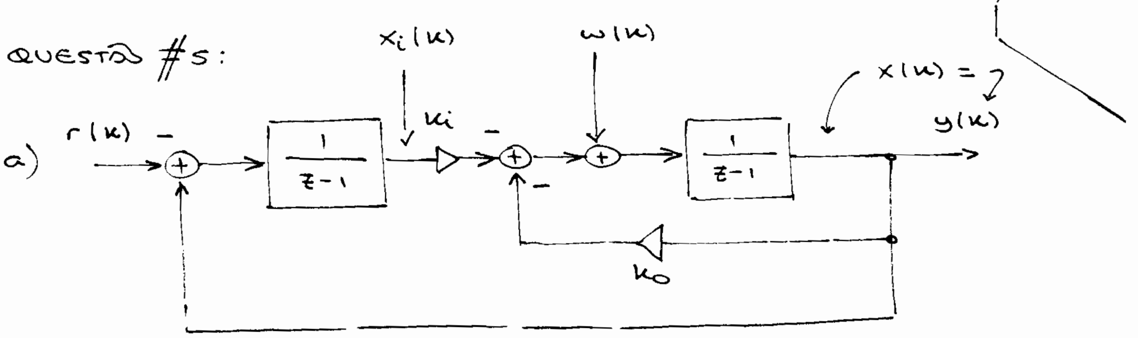
	1	-2	1			
0.7	1				1	1
	0.7	1		-0.7	1.1	
		0.7	1		-0.7	1.1
			0.7	1	-0.7	1.1
				1.1	0.4	-0.7

1	-1.3	-0.4	0.7
	1.1	0.4	-0.7
1	-0.2	0	0

$\longrightarrow z^3 - 0.2z^2 = (z-0.2)z^2$

QUESTÃO #5:

a)



$$x_i(k+1) = x_i(k) + x(k) - r(k)$$

$$x(k+1) = x(k) + u(k) + \omega(k) = -k_i x_i(k) + (1-k_o)x(k) + \omega(k)$$

$$u(k) = -k_i x_i(k) - k_o x(k)$$

$$\begin{bmatrix} x_i(k+1) \\ x(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -k_i & 1-k_o \end{bmatrix}\begin{bmatrix} x_i(k) \\ x(k) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} r(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\omega(k)$$

$$y(k) = [0\ \ 1]\begin{bmatrix} x_i(k) \\ x(k) \end{bmatrix}$$

b) $$z = e^{sT} = \underbrace{e^{-0.3466}}_{\frac{\sqrt{2}}{2}}\left(\underbrace{\cos(0.7854)}_{\frac{\sqrt{2}}{2}} \pm j\underbrace{\sin(0.7854)}_{\frac{\sqrt{2}}{2}}\right) = 0.5 \pm 0.5j$$

$$\alpha_c(z) = (z-0.5-0.5j)(z-0.5+0.5j) = z^2 - z + 0.5$$

c) $$\phi_i - \Gamma_i K = \begin{bmatrix} 1 & 1 \\ -K_i & 1-K_0 \end{bmatrix}$$

$$|zI - \phi_i + \Gamma_i K| = \begin{vmatrix} z-1 & -1 \\ K_i & z+K_0-1 \end{vmatrix} = z^2 + \underbrace{(K_0-2)}_{K_0=1} z + \underbrace{1 - K_0 + K_i}_{K_i = 0.5} \quad (1)$$

$$K = [0.5 \quad 1]$$

d) SUBSTITUINDO K_i E K_0 DO ITEM (c) NAS EQUAÇÕES DE ESTADO:

$$\begin{bmatrix} x_i(k+1) \\ x(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -0.5 & 0 \end{bmatrix} \begin{bmatrix} x_i(k) \\ x(k) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} r(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_i(k) \\ x(k) \end{bmatrix}$$

- $$\frac{Y(z)}{R(z)} = [0 \quad 1] \begin{bmatrix} z-1 & -1 \\ 0.5 & z \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = [0 \quad 1] \frac{\begin{bmatrix} z & 1 \\ -0.5 & z-1 \end{bmatrix}}{z^2 - z + 0.5} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \frac{0.5}{z^2 - z + 0.5}$$

$$\left(\lim_{z \to 1} \frac{Y(z)}{R(z)} = 1 \right)$$

- $$\frac{Y(z)}{W(z)} = [0 \quad 1] \frac{\begin{bmatrix} z & 1 \\ -0.5 & z-1 \end{bmatrix}}{z^2 - z + 0.5} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{z-1}{z^2 - z + 0.5}$$

$$\left(\lim_{z \to 1} \frac{Y(z)}{W(z)} = 0 \right)$$

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CONTROLE II — SEGUNDO SEMESTRE DE 2006

GABARITO DA PROVA PARCIAL # 2 — 01/12/2006 — MODELO B

QUESTÃO # 1 :

a) $$D(z) = \frac{\frac{z-1}{T}}{\frac{z-1}{T} + 1} = \frac{z-1}{z+T-1}$$

b) $$D(z) = \frac{\frac{z-1}{Tz}}{\frac{z-1}{Tz} + 1} = \frac{z-1}{(T+1)z-1}$$

c) $$D(z) = \frac{z-1}{z-e^{-T}}$$

$e^{0T} = 1$ ($s=0 \rightarrow z=1$) (ZERO)

$e^{-1T} = e^{-T}$ ($s=-1 \rightarrow z=e^{-T}$) (PÓLO)

d)

RESULTADOS PARA T = 0.1

FORWARD EULER : $D(z) = \frac{z-1}{z-0.9} \longrightarrow Y(z) = \frac{z}{z-1} D(z) = \frac{z}{z-0.9}$

$y(k) = 0.9^k u(k)$

BACKWARD EULER : $D(z) = \frac{z-1}{1.1z-1} = \frac{0.91(z-1)}{z-0.91} \longrightarrow Y(z) = \frac{0.91\,z}{z-0.91}$

$y(k) = 0.91^{k+1} u(k)$

MAPEAMENTO DOS PÓLOS E ZEROS :

$D(z) = \frac{z-1}{z-0.90} \longrightarrow Y(z) = \frac{z}{z-0.9}$

$y(k) = 0.9^k u(k)$

RESULTADOS PARA T = 0.5

FORWARD EULER : $D(z) = \frac{z-1}{z-0.5} \longrightarrow Y(z) = \frac{z}{z-0.5}$

$y(k) = 0.5^k u(k)$

BACKWARD EULER : $D(z) = \frac{z-1}{1.5z-1} = \frac{0.67(z-1)}{z-0.67} \longrightarrow Y(z) = \frac{0.67\,z}{z-0.67}$

$y(k) = 0.67^{k+1} u(k)$

MAPEAMENTO DOS PÓLOS E ZEROS :

$D(z) = \frac{z-1}{z-0.61} \longrightarrow Y(z) = \frac{z}{z-0.61}$

$y(k) = 0.61^k u(k)$

PARA T = 0.1 , TODOS OS TRÊS MÉTODOS GERAM UMA SEQÜÊNCIA DE VALORES MUITO PRÓXIMA DA SEQÜÊNCIA DE AMOSTRAS DA RESPOSTA AO DEGRAU DE D(S), QUE É $y(t) = e^{-t}u(t)\Big|_{t=0.1k}$. NOTE QUE, NESTE CASO, O MÉTODO DO MAPEAMENTO DE PÓLOS E ZEROS TEM ERRO IGUAL A ZERO. QUANDO T AUMENTA PARA T = 0.5 , O ERRO DOS MÉTODOS DE EULER AUMENTA (O PIOR DESEMPENHO É DO MÉTODO FORWARD EULER), MAS O MAPEAMENTO DE PÓLOS E ZEROS MANTÉM ERRO IGUAL A ZERO NA GERAÇÃO DAS AMOSTRAS DA RESPOSTA AO DEGRAU y(t).

QUESTÃO #2:

a) $e^{Ft} = \mathcal{L}^{-1}\left[\begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}^{-1}\right] = \mathcal{L}^{-1}\begin{bmatrix} \frac{s}{(s+j)(s-j)} & \frac{1}{(s+j)(s-j)} \\ \frac{-1}{(s+j)(s-j)} & \frac{s}{(s+j)(s-j)} \end{bmatrix}$

$\frac{1}{(s+j)(s-j)} = \frac{j/2}{s+j} - \frac{j/2}{s-j} \longrightarrow \left(\frac{j}{2}e^{-jt} - \frac{j}{2}e^{jt}\right)u(t) = (\sin t)u(t)$

$\frac{s}{(s+j)(s-j)} = \frac{1/2}{s+j} + \frac{1/2}{s-j} \longrightarrow \left(\frac{1}{2}e^{-jt} + \frac{1}{2}e^{jt}\right)u(t) = (\cos t)u(t)$

$\phi = e^{FT} = \begin{bmatrix} \cos T & \sin T \\ -\sin T & \cos T \end{bmatrix}$

$\Gamma = \int_0^T e^{Ft}G\,dt = \begin{bmatrix} \sin t \Big|_0^T \\ \cos t \Big|_0^T \end{bmatrix} = \begin{bmatrix} \sin T \\ \cos T - 1 \end{bmatrix}$

b) $\mathcal{C} = \begin{bmatrix} \sin T & 2\sin T\cos T - \sin T \\ \cos T - 1 & -\sin^2 T + \cos^2 T - \cos T \end{bmatrix}$

$+ 2\sin T\cos T$

$+ \sin T\cos T$

$\det \mathcal{C} = -\sin^3 T + \sin T\cos^2 T - \sin T\cos T - 2\sin T\cos^2 T - \sin T$

$\det \mathcal{C} = -\sin T\,(\underbrace{\sin^2 T + \cos^2 T}_{1} + \cos T + 1 - 3\cos T)$

$\det \mathcal{C} = 0 \iff \sin T\,(2 - 2\cos T) = 0$

(SUBCONJUNTO DE $\sin T = 0$)

$\longrightarrow \cos T = 1$ se $T = 2k\pi,\ k \in \mathbb{Z}^+$.

$\sin T = 0 \longrightarrow$ SISTEMA NÃO-CONTROLÁVEL SE $T = k\pi,\ k \in \mathbb{Z}^+$.

c) $\Theta = \begin{bmatrix} a & b \\ a\cos T - b\sin T & a\sin T + b\cos T \end{bmatrix}$

$\det \Theta = a^2 \sin T + \cancel{ab\cos T} - \cancel{ab\cos T} + b^2 \sin T$

$\det \Theta = 0 \iff (a^2 + b^2)\sin T = 0$

$\hookrightarrow$ SISTEMA NÃO-OBSERVÁVEL SE $(a=0$ e $b=0)$ ou $(T = k\pi,\ k \in \mathbb{Z}^+)$

d) $|zI - \phi| = \begin{vmatrix} z - \cos T & -\sin T \\ \sin T & z - \cos T \end{vmatrix} = z^2 - 2\cos T + 1 = 0$

PÓLOS: $\frac{2\cos T \pm \sqrt{4\cos^2 T - 4}}{2} = \cos T \pm \sqrt{-\sin^2 T} = \cos T \pm j\sin T$

OS PÓLOS ESTÃO SOBRE O CÍRCULO UNITÁRIO, E PORTANTO O SISTEMA É MARGINALMENTE ESTÁVEL DESDE QUE $T \neq k\pi,\ k \in \mathbb{Z}^+$ (PÓLO DUPLO EM $z=1$ OU $z=-1$).

QUESTÃO #3:

a) SEJA $P = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = T^{-1}$. ENTÃO $T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$.

$$\phi_z = T^{-1}\phi T = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.18 & 0.64 \\ -0.81 & 1.62 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0.18 & 1 \\ -0.81 & 0 \end{bmatrix} = \begin{array}{c} \phi_{aa} \quad \phi_{ab} \\ \left[\begin{array}{c|c} 1.8 & 1 \\ \hline -0.81 & 0 \end{array}\right] \\ \phi_{ba} \quad \phi_{bb} \end{array}$$

$$\Gamma_z = T^{-1}\Gamma = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \left[\begin{array}{c} 0 \\ \hline 1 \end{array}\right] \begin{array}{l} \Gamma_a \\ \Gamma_b \end{array}$$

$$H_z = HT = [1 \;\; 0]$$

$$|zI - \phi_{bb} + L\phi_{ab}| = z + L = z + 0.3 \longrightarrow L = 0.3$$

b) $$\underbrace{\hat{x}_b(k+1)}_{\hat{z}_2(k+1)} = \underbrace{(\phi_{bb} - L\phi_{ab})}_{-0.3}\underset{\hat{z}_2(k)}{\underset{\downarrow}{\hat{x}_b(k)}} + \underbrace{(\phi_{ba} - L\phi_{aa})}_{-1.35}\underset{y(k)}{\underset{\downarrow}{x_a(k)}} + \underbrace{(\Gamma_b - L\Gamma_a)}_{1.0}u(k) + \underset{0.3}{\underset{\downarrow}{L}}\underbrace{x_a(k+1)}_{y(k+1)}$$

$$u(k) = -K\hat{x} = -KT\hat{z} = -0.4y(k) - 0.4\hat{z}_2(k)$$

$$KT = [0.4 \;\; -0.4]\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = [0.4 \;\; 0.4]$$

ENTÃO: $\hat{z}_2(k+1) = -0.7\hat{z}_2(k) - 1.75y(k) + 0.3y(k+1)$

c) $$\hat{z}_2(z)(z+0.7) = Y(z)(0.3z - 1.75)$$

$$\hat{z}_2(z) = \left(\frac{0.3z - 1.75}{z + 0.7}\right)Y(z)$$

$$U(z) = -0.4Y(z) - 0.4\hat{z}_2(z) = -0.4\left(1 + \frac{0.3z - 1.75}{z+0.7}\right)Y(z) = \left(\frac{-0.52z + 0.42}{z + 0.7}\right)Y(z)$$

$$D(z) = \frac{-0.52z + 0.42}{z + 0.7}$$

d) $$G(z) = [1 \;\; 0]\begin{bmatrix} z-1.8 & -1 \\ 0.81 & z \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{[1 \;\; 0]\begin{bmatrix} z & 1 \\ -0.81 & z-1.8 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{z^2 - 1.8z + 0.81} = \frac{1}{z^2 - 1.8z + 0.81}$$

$$\frac{G(z)}{1 - G(z)D(z)} = \frac{\frac{1}{z^2 - 1.8z + 0.81}}{1 + \frac{(0.52z - 0.42)}{(z^2 - 1.8z + 0.81)(z + 0.7)}} = \frac{\overbrace{z + 0.7}^{\gamma(z)}}{\underbrace{(z - 0.7)^2}_{\alpha_c(z)}\underbrace{(z + 0.3)}_{\alpha_e(z)}}$$

	1	-1.8	0.81		
0.7	1				
	0.7	1			
		0.7	1		
			0.7	1	
	1	-1.1	-0.45	0.567	
			0.52	-0.42	
	1	-1.1	0.07	0.147	

$$z^3 - 1.1z^2 + 0.07z + 0.147 \;\big|\; z + 0.3$$
$$z^3 + 0.3z^2 \qquad\qquad z^2 - 1.4z + 0.49$$
$$-1.4z^2 + 0.07z$$
$$-1.4z^2 - 0.42z$$
$$0.49z$$

NOTE QUE: $|zI - \phi_z + \Gamma_z K_z| =$

$$= \begin{vmatrix} z - 1.8 & -1 \\ 1.21 & z + 0.4 \end{vmatrix} = z^2 - 1.4z + 0.49$$

QUESTÃO #4:

a) $\phi - LH - \Gamma K = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} [1 \quad 0] - \begin{bmatrix} 1.25 & -0.75 \\ -1.25 & 0.75 \end{bmatrix} = \begin{bmatrix} -1.25 & -0.25 \\ 1.25 & -0.75 \end{bmatrix}$

$\hat{x}(k+1) = \begin{bmatrix} -1.25 & -0.25 \\ 1.25 & -0.75 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} y(k)$

$u(k) = -\begin{bmatrix} 1.25 & -0.75 \end{bmatrix} \hat{x}(k)$

b) INSTANTE $t = kT$: k

ENVIAR $u(k)$ P/ CONVERSOR D/A

AMOSTRAR $y(k)$

$\hat{x}(k+1) = \begin{bmatrix} -1.25 & -0.25 \\ 1.25 & -0.75 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} y(k)$

$u(k+1) = -[1.25 \quad -0.75] \hat{x}(k)$

⋮

INSTANTE $t = (k+1)T$: $k+1$

⋮

c) $D(z) = -[1.25 \quad -0.75] \begin{bmatrix} z+1.25 & 0.25 \\ -1.25 & z+0.75 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$D(z) = \dfrac{-\frac{1}{4}[5 \quad -3] \begin{bmatrix} z+0.75 & -0.25 \\ 1.25 & z+1.25 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}}{z^2 + 2z + 1.25} = \dfrac{-\frac{1}{4}[5 \quad -3] \begin{bmatrix} 2z + 1.25 \\ z + 3.75 \end{bmatrix}}{z^2 + 2z + 1.25}$

$D(z) = \dfrac{-1.75z + 1.25}{z^2 + 2z + 1.25} \longrightarrow$ PÓLOS DO COMPENSADOR ($\gamma(z)$): $-1 \pm 0.5j$

d) MALHA FECHADA:

$G(z) = [1 \quad 0] \begin{bmatrix} z-2 & 1 \\ -1 & z \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \dfrac{[1 \quad 0] \begin{bmatrix} z & -1 \\ 1 & z-2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}}{z^2 - 2z + 1} = \dfrac{z+1}{z^2 - 2z + 1}$

$\dfrac{G(z)}{1 - G(z)D(z)} = \dfrac{\dfrac{z+1}{z^2-2z+1}}{1 + \dfrac{(z+1)(1.75z - 1.25)}{(z^2 - 2z + 1)(z^2 + 2z + 1.25)}} = \dfrac{\overset{b(z)}{(z+1)}\overset{\gamma(z)}{(z^2 + 2z + 1.25)}}{\underset{\alpha_c(z)}{z^2} \cdot \underset{\alpha_e(z)}{z^2}}$

	1	-2	1			
1.25	2	1				
	1.25	2	1			
		1.25	2	1		
			1.25	2	1	
				1.25	2	1

1	1	
-1.25	1.75	
-1.25	1.75	
-1.25	1.75	
1.75	0.5	-1.25

1	0	-1.75	-0.5	1.25
		1.75	0.5	-1.25
1	0	0	0	0

$\longrightarrow z^4$

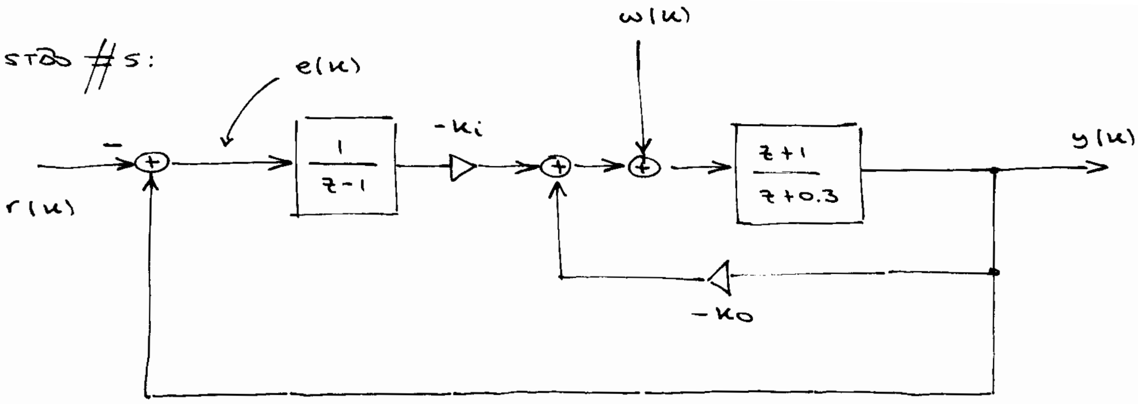
JUSTIFICANDO:

$|zI - \phi + \Gamma K| = \begin{vmatrix} z - 0.75 & 0.25 \\ -2.25 & z + 0.75 \end{vmatrix}$ ✓

$|zI - \phi + LH| = \begin{vmatrix} z & 1 \\ 0 & z \end{vmatrix} = z^2$ ✓

QUESTÃO #5:

a)



b) $w(z) = 0 \implies Y(z) = \left(\frac{z+1}{z+0.3}\right) U(z)$

$$U(z) = -k_i \frac{(Y(z) - R(z))}{z-1} - k_o Y(z)$$

$$Y(z)\left(\frac{z+0.3}{z+1} + \frac{k_i}{z-1} + k_o\right) = R(z) \cdot \frac{k_i}{z-1}$$

$$\frac{Y(z)}{R(z)} = \frac{\frac{k_i}{z-1}}{\frac{z+0.3}{z+1} + \frac{k_i}{z-1} + k_o} = \frac{k_i(z+1)}{(z+0.3)(z-1) + k_i(z+1) + k_o(z^2-1)}$$

$$\frac{Y(z)}{R(z)} = \frac{k_i(z+1)}{(k_o+1)z^2 + (k_i-0.7)z + k_i - k_o - 0.3}$$

$$\frac{Y(z)}{R(z)} = \frac{\frac{k_i}{k_o+1}(z+1)}{z^2 + \frac{(k_i-0.7)z}{k_o+1} + \frac{k_i-k_o-0.3}{k_o+1}}$$

c) $\alpha_c(z) = (z-0.5+0.5j)(z-0.5-0.5j) = z^2 - z + 0.5$

$$k_i - 0.7 = -(k_o+1) \implies k_i + k_o = -0.3$$

$$k_i - k_o - 0.3 = 0.5k_o + 0.5 \implies k_i - 1.5k_o = 0.8$$

$$(-) \qquad 2.5k_o = -1.1$$

$$k_o = -0.44 \longrightarrow k_i = 0.14$$

<u>OBS.:</u> NOTE QUE ASSIM $\frac{Y(z)}{R(z)} = \frac{0.25(z+1)}{z^2 - z + 0.5}$

d) $R(z) = 0 \implies \left(-k_i \frac{Y(z)}{z-1} - k_o Y(z) + W(z)\right)\left(\frac{z+1}{z+0.3}\right) = Y(z)$

$$W(z) = \left(\frac{z+0.3}{z+1}\right)Y(z) + \frac{k_i}{z-1}Y(z) + k_o Y(z)$$

$$\frac{W(z)}{Y(z)} = \frac{(z+0.3)(z-1) + k_i(z+1) + k_o(z^2-1)}{(z+1)(z-1)}$$

$$\frac{Y(z)}{W(z)} = \frac{z^2-1}{z^2 + \frac{k_i-0.7}{k_o+1}z + \frac{k_i-k_o-0.3}{k_o+1}}$$

SE $k_o = -0.44$ E $k_i = 0.14 \implies \frac{Y(z)}{W(z)} = \frac{z^2-1}{z^2 - z + 0.5}$