CONTROLE II —— PRIMEIRO SEMESTRE DE 2006

GABARITO DA PROVA PARCIAL #2 —— 19/06/2006

QUESTÃO #1:

a) zero: $e^{-0.4} = 0.67$

$$D(z) = \frac{k(z-0.67)}{z^2-1.84z+1}$$

pólos: $e^{\pm 0.4j} = 0.92 \pm 0.39j$

$$\lim_{z\to 1} D(z) = \lim_{s\to 0} D(s) \Rightarrow 2.06k = 2 \Rightarrow k = 0.97$$

b)
$$D(z) = \frac{2\left(\frac{2}{T}\frac{(z-1)}{(z+1)}+1\right)}{\left(\frac{2}{T}\frac{(z-1)}{(z+1)}\right)^2+1} = \frac{2\left(2T(z-1)(z+1)+T^2(z+1)^2\right)}{4(z-1)^2+T^2(z+1)^2}$$

$$D(z) = \frac{2\left(z^2(T^2+2T)+2T^2z+T^2-2T\right)}{(T^2+4)z^2+(2T^2-8)z+T^2+4} = \frac{1.92(z^2+0.33z-0.67)}{4.16(z^2-1.85z+1)}$$

$$D(z) = \frac{0.46(z+1)(z-0.67)}{z^2-1.85z+1}$$

c)
$$\frac{D(s)}{s} = \frac{2(s+1)}{s(s^2+1)} = \frac{A}{s}+\frac{B}{s+j}+\frac{C}{s-j}$$

$$A = \left.\frac{2(s+1)}{s^2+1}\right|_{s=0} = 2 \quad ; \quad B = \left.\frac{2(s+1)}{s(s-j)}\right|_{s=-j} = \frac{2(1-j)}{(-j)(-2j)} = -1+j \quad ; \quad C = -1-j$$

$$y(t) = \left(2+(-1+j)e^{-jt}+(-1-j)e^{jt}\right)u(t)$$

$$y(k) = \left(2+(-1+j)e^{-jTk}+(-1-j)e^{jTk}\right)u(k)$$

$$Y(z) = \frac{2z}{z-1}+(-1+j)\frac{z}{z-e^{-jT}}+(-1-j)\frac{z}{z-e^{jT}}$$

$$D(z) = 2+(z-1)\underbrace{\left(\frac{-1+j}{z-e^{-jT}}+\frac{-1-j}{z-e^{jT}}\right)}$$

$$\frac{z(-1+\cancel{j}-1-\cancel{j})+e^{jT}-je^{jT}+e^{-jT}+je^{-jT}}{z^2-(e^{jT}+e^{-jT})z+1}$$

$$\frac{-2z+2\cos(T)+2\sin(T)}{z^2-2\cos(T)+1}$$

$$\frac{-2z+2.62}{z^2-1.84z+1}$$

NOTE QUE:

$e^{jT}+e^{-jT} = 2\cos(T)$

E

$\sin(T) = \frac{e^{jT}-e^{-jT}}{2j}$

$2\sin(T) = je^{-jT}-je^{jT}$

$$D(z) = 2+\frac{(z-1)(-2z+2.62)}{z^2-1.84z+1} = \frac{\cancel{2z^2}-3.68z+2-\cancel{2z^2}+2.62z+2z-2.62}{z^2-1.84z+1}$$

$$D(z) = \frac{0.94z-0.62}{z^2-1.84z+1}$$

$$D(z) = \frac{0.94(z-0.66)}{z^2-1.84z+1}$$

QUESTÃO # 2:

a) $G(s) = \frac{1}{s} + \frac{(-1)}{s+1}$

FCM: $F = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$; $G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $H = [1 \quad -1]$

$$\Phi = e^{FT} = \begin{bmatrix} 1 & 0 \\ 0 & e^{-T} \end{bmatrix}$$

$$\Gamma = \int_0^T \begin{bmatrix} 1 \\ e^{-\sigma} \end{bmatrix} d\sigma = \begin{bmatrix} T \\ 1-e^{-T} \end{bmatrix}$$

$$G(z) = H(zI - \Phi)^{-1}\Gamma = [1 \quad -1] \begin{bmatrix} z-1 & 0 \\ 0 & z-e^{-T} \end{bmatrix}^{-1} \begin{bmatrix} T \\ 1-e^{-T} \end{bmatrix}$$

$$= \frac{[1 \quad -1] \begin{bmatrix} z-e^{-T} & 0 \\ 0 & z-1 \end{bmatrix} \begin{bmatrix} T \\ 1-e^{-T} \end{bmatrix}}{z^2-(1+e^{-T})z+e^{-T}}$$

$$G(z) = \frac{z(T+e^{-T}-1) + 1 - e^{-T} - Te^{-T}}{z^2-(1+e^{-T})z+e^{-T}}$$

b) $\Phi_{cc} = \begin{bmatrix} e^{-T}+1 & -e^{-T} \\ 1 & 0 \end{bmatrix}$; $\Gamma_{cc} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $H_{cc} = [T+e^{-T}-1 \quad 1-e^{-T}-Te^{-T}]$

c) $z = e^{-2\pm 2j} = -0.06 \pm 0.12j \longrightarrow \alpha_c(z) = z^2 + 0.12z + 0.02$

FCC: $k_1 - (1+e^{-T}) = +0.12 \longleftarrow$ POR INSPEÇÃO DO DENOMINADOR DE $G(z)$.

$k_1 - 1.82 = 0.12$

$k_1 = 1.94$

$k_2 + e^{-T} = 0.02 \longleftarrow$ POR INSPEÇÃO DO DENOMINADOR DE $G(z)$.

$k_2 + 0.82 = 0.02$

$k_2 = -0.80$

PORTANTO: $K = [1.94 \quad -0.80]$

d) $$\frac{Y(z)}{R(z)} = \frac{z(T+e^{-T}-1) + (1-e^{-T}-Te^{-T})}{z^2+0.12z+0.02} = \frac{0.02z+0.02}{z^2+0.12z+0.02}$$

$(T+e^{-T}-1 = 0.02$ E $1-e^{-T}-Te^{-T} = 0.02)$

GANHO DC: $$\lim_{z\to 1} \frac{Y(z)}{R(z)} = \frac{0.02+0.02}{1+0.12+0.02} = 0.035$$

QUESTÃO #3:

a) $\Phi = \begin{bmatrix} 1 & 1 \\ -0.5 & 0 \end{bmatrix}$; $\Gamma = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$; $H = [1 \;\; 0]$; $\quad x(k+1) = \Phi x(k) + u(k)$

$y(k) = Hx(k)$

b) $|zI - \Phi + \Gamma K| = \begin{vmatrix} z-1+k_1 & -1+k_2 \\ 0.5+0.5k_1 & z+0.5k_2 \end{vmatrix} = z^2 + (k_1 + 0.5k_2 - 1)z + \cancel{0.5k_1k_2} - 0.5k_2$

$+ 0.5 + 0.5k_1 - 0.5k_2 - \cancel{0.5k_1k_2}$

$= z^2 + (k_1 + 0.5k_2 - 1)z + 0.5k_1 - k_2 + 0.5$

$\alpha_c(z) = (z - 0.25 - 0.25j)(z - 0.25 + 0.25j) = z^2 - 0.5z + 0.125$

$|zI - \Phi + \Gamma K| = \alpha_c(z) \implies \begin{cases} k_1 + 0.5k_2 = 0.5 \\ 0.5k_1 - k_2 = -0.375 \longrightarrow k_1 - 2k_2 = -0.75 \end{cases}$

$2.5k_2 = 1.25 \longrightarrow k_2 = 0.5$

$k_1 = -0.75 + 2k_2 \longrightarrow k_1 = 0.25$

$K = [0.25 \;\; 0.5]$

c)
$$\begin{array}{cc} \Phi_{aa} & \Phi_{ab} \end{array} \qquad\qquad \Gamma_a$$
$$\left[\begin{array}{c|c} 1 & 1 \\ \hline -0.5 & 0 \end{array}\right] \qquad \left[\begin{array}{c} 1 \\ \hline 0.5 \end{array}\right] \Gamma_b$$
$$\begin{array}{cc} \Phi_{ba} & \Phi_{bb} \end{array}$$

$z + L = z + 0.1$

$z - \Phi_{bb} + L\Phi_{ab} = z + L \longrightarrow L = 0.1$

$\alpha_e(z) = z + 0.1$

d) $\hat{x}_2(k+1) = (\Phi_{ba} - L\Phi_{aa})y(k) + Ly(k+1) + (\Phi_{bb} - L\Phi_{ab})\hat{x}_2(k) + (\Gamma_b - L\Gamma_a)u(k)$

$\Phi_{ba} - L\Phi_{aa} = -0.6 \qquad u(k) = -0.25y(k) - 0.5\hat{x}_2(k)$

$\Phi_{bb} - L\Phi_{ab} = -0.1$

$\Gamma_b - L\Gamma_a = 0.4$

ENTÃO: $\hat{x}_2(k+1) = -0.6y(k) + 0.1y(k+1) - 0.1\hat{x}_2(k) - 0.1y(k) - 0.2\hat{x}_2(k)$

$\hat{x}_2(k+1) = -0.3\hat{x}_2(k) - 0.7y(k) + 0.1y(k+1)$

e) $X_2(z) = \dfrac{(0.1z - 0.7)}{z + 0.3} \cdot Y(z)$

$U(z) = -0.25Y(z) - 0.5\dfrac{(0.1z - 0.7)}{z+0.3}Y(z) = \dfrac{(-0.25z - 0.075 - 0.05z + 0.35)\,Y(z)}{z + 0.3}$

$D(z) = \dfrac{U(z)}{Y(z)} = \dfrac{-0.3z + 0.275}{z + 0.3}$

OBS.: $\dfrac{Y(z)}{R(z)} = \dfrac{\dfrac{z+0.5}{z^2 - z + 0.5}}{1 - \dfrac{(z+0.5)(-0.3z + 0.275)}{(z^2 - z + 0.5)(z + 0.3)}}$

$\dfrac{Y(z)}{R(z)} = \dfrac{(z+0.5)(z+0.3)}{z^3 - z^2 + 0.5z + 0.3z^2 - 0.3z + 0.15 + 0.3z^2 - 0.275z + 0.15z - 0.1375}$

$\dfrac{Y(z)}{R(z)} = \dfrac{z^2 + 0.8z + 0.15}{z^3 - 0.4z^2 + 0.075z + 0.0125} = \dfrac{(z+0.5)(z+0.3)}{(z+0.1)(z^2 - 0.5z + 0.125)}$

$\alpha_e(z) \qquad \alpha_c(z)$ ✓

QUESTÃO #4:

a) $x(k+1) = \Phi x(k) + \Gamma u(k)$, ONDE $\Phi = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $\Gamma = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $H = [1 \ \ 0 \ \ 0]$

$y(k) = Hx(k)$

NO FCO, OBTÉM-SE $|zI - \Phi + LH|$ POR INSPEÇÃO:

$|zI - \Phi + LH| = z^3 + \ell_1 z^2 + \ell_2 z + (-1 + \ell_3)$

$\alpha_e(z) = z^3$

$\alpha_e(z) = |zI - \Phi + LH| \Rightarrow \ell_1 = 0$, $\ell_2 = 0$ E $\ell_3 = 1$. $\quad L = L_p = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

b) $L_a = \Phi^{-1} L_p = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \cancel{L_a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} \quad \left(L_a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$

c) EQUAÇÃO DO ESTIMADOR DE PREDIÇÃO (SUBSTITUINDO $u = -K\hat{x}$):

$\hat{x}(k+1) = (\Phi - LH - \Gamma K)\,\hat{x}(k) + Ly(k)$

$u(k+1) = -K\hat{x}(k+1)$

COMPARANDO COM O PSEUDO-CÓDIGO:

$A = \Phi - LH - \Gamma K = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [1 \ \ 0 \ \ 0] - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [1 \ \ 0 \ \ 0] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$

$B = L = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$C = -K \Rightarrow C = [-1 \ \ 0 \ \ 0]$

d) EQUAÇÃO DO ESTIMADOR ATUALIZADO (SUBSTITUINDO $u = -K\hat{x}$):

$\hat{x}(k) = \underbrace{(\Phi - LH\Phi - \Gamma K + LH\Gamma K)\,\hat{x}(k-1)}_{\breve{x}(k)} + Ly(k)$

$u(k) = -K\hat{x}(k)$

$\breve{x}(k+1) = (\Phi - LH\Phi - \Gamma K + LH\Gamma K)\,\hat{x}(k)$

COMPARANDO COM O PSEUDO-CÓDIGO:

$X = L = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $Y = -K \Rightarrow Y = [-1 \ \ 0 \ \ 0]$

$Z = \underbrace{\Phi - LH\Phi} - \underbrace{\Gamma K + LH\Gamma K} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\Phi - LH\Phi = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [0 \ \ 1 \ \ 0] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\Gamma - LH\Gamma = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \underbrace{[1 \ \ 0 \ \ 0] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{0} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$(\Gamma - LH\Gamma)K = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [1 \ \ 0 \ \ 0] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

QUESTÃO # 5:

a) $x_{i_1}(k+1) = x_{i_1}(k) + e(k) \longrightarrow x_{i_1}(k+1) = x_{i_1}(k) + x(k) - r(k)$

$x_{i_2}(k+1) = x_{i_2}(k) + x_{i_1}(k)$

$x(k+1) = 0.2x(k) + u(k) + w(k) \longrightarrow x(k+1) = 0.2x(k) + w(k) - k_{i_1}x_{i_1}(k)$

$- k_{i_2}x_{i_2}(k) - k_0 x(k)$

$$\begin{bmatrix} x_{i_1}(k+1) \\ x_{i_2}(k+1) \\ \cancel{x(k)}\, x(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -k_{i_1} & -k_{i_2} & 0.2-k_0 \end{bmatrix}}_{\Phi_i - \Gamma_i K} \begin{bmatrix} x_{i_1}(k) \\ x_{i_2}(k) \\ x(k) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w(k)$$

(*)

(*): OBS.: $y(k) = [0 \quad 0 \quad 1] \begin{bmatrix} x_{i_1}(k) \\ x_{i_2}(k) \\ x(k) \end{bmatrix}$

b) $|zI - \Phi_i + \Gamma_i K| = \begin{vmatrix} z-1 & 0 & -1 \\ -1 & z-1 & 0 \\ k_{i_1} & k_{i_2} & z-0.2+k_0 \end{vmatrix} = (z^2-2z+1)(z-0.2+k_0) + k_{i_2} + k_{i_1}z - k_{i_1}$

$= z^3 - 2z^2 + z - 0.2z^2 + k_0z^2 + 0.4z - 2k_0z - 0.2 + k_0 + k_{i_2} + k_{i_1}z - k_{i_1}$

$= z^3 + (k_0 - 2.2)z^2 + (k_{i_1} - 2k_0 + 1.4)z + k_0 + k_{i_2} - k_{i_1} - 0.2 = 0$

DEAD · BEAT: $\alpha_c(z) = z^3 \implies k_0 = 2.2$

$k_{i_1} - 4.4 + 1.4 = 0 \implies k_{i_1} = 3$

$2.2 + k_{i_2} - 3 - 0.2 = 0 \implies k_{i_2} = 1$

PORTANTO $K = [3 \quad 1 \quad 2.2]$

c) USANDO $K = [3 \quad 1 \quad 2.2]$: $\Phi_i - \Gamma_i K = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ -3 & -1 & -2 \end{bmatrix}$

(1,3)

$$\frac{Y(z)}{R(z)} = [0 \quad 0 \quad 1] \begin{bmatrix} z-1 & 0 & -1 \\ \boxed{\begin{matrix} -1 & z-1 \\ 3 & 1 \end{matrix}} & & \begin{matrix} 0 \\ z+2 \end{matrix} \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \frac{(-1-3z+3)(-1)}{z^3} \implies \frac{Y(z)}{R(z)} = \frac{3z-2}{z^3}$$

d) $$\frac{Y(z)}{W(z)} = [0 \quad 0 \quad 1] \begin{bmatrix} \boxed{\begin{matrix} z-1 & 0 \\ -1 & z-1 \end{matrix}} & \begin{matrix} -1 \\ 0 \end{matrix} \\ \begin{matrix} 3 & 1 \end{matrix} & z+2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{(z-1)^2}{z^3}$$

(3,3)

$$\frac{Y(z)}{W(z)} = \frac{(z-1)^2}{z^3}$$