Universidade Federal do Rio de Janeiro
Escola Politécnica

Data: 21/11/05

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Aluno: GABARITO DA 2ª PROVA PARCIAL

Disciplina: CONTROLE LINEAR II — 2005 2º SEMESTRE
Turma:

Professor: GABRIEL

1) a) ZERO: $e^{-0.2} = 0.82$ PÓLOS: $e^{-0.2 \pm 0.2j} = 0.82(\cos 0.2 \pm j \sin 0.2) = 0.8 \pm 0.16j$

$$D(z) = \frac{z - 0.82}{(z-0.8-0.16j)(z-0.8+0.16j)} = \frac{z-0.82}{z^2 - 1.6z + 0.67}$$

b) $$D(z) = \frac{\frac{z-1}{T} + 1}{\left(\frac{z-1}{T}\right)^2 + 2\left(\frac{z-1}{T}\right) + 2} = \frac{Tz - T + T^2}{z^2 - 2z + 1 + 2Tz - 2T + 2T^2} = \frac{Tz + T^2 - T}{z^2 + (2T-2)z + 2T^2 - 2T + 1}$$

$$D(z) = \frac{0.2z - 0.16}{z^2 - 1.6z + 0.68}$$

ZERO: 0.8

PÓLOS: $0.8 \pm 0.2j$

c) $$D(z) = \frac{\left(\frac{z-1}{Tz}\right) + 1}{\left(\frac{z-1}{Tz}\right)^2 + 2\left(\frac{z-1}{Tz}\right) + 2} = \frac{(z-1)Tz + (Tz)^2}{z^2 - 2z + 1 + 2Tz^2 - 2Tz + 2T^2z^2} = \frac{(T^2+T)z^2 - Tz}{(2T^2+2T+1)z^2 + (-2-2T)z + 1}$$

$$D(z) = \frac{0.24z^2 - 0.2z}{1.48z^2 - 2.4z + 1}$$

ZEROS: 0 E 0.83

PÓLOS: $0.81 \pm 0.14j$

2) a) $$G(s) = \frac{2(s+\sigma)}{(s+\sigma+j\omega)(s+\sigma-j\omega)} = \frac{1}{s+\sigma+j\omega} + \frac{1}{s+\sigma-j\omega}$$

$$F = \begin{bmatrix} -\sigma - j\omega & 0 \\ 0 & -\sigma + j\omega \end{bmatrix} \quad G = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad H = [1 \;\; 1]$$

$$\Phi = e^{FT} = \begin{bmatrix} e^{(-\sigma-j\omega)T} & 0 \\ 0 & e^{(-\sigma+j\omega)T} \end{bmatrix} = \begin{bmatrix} e^{pT} & 0 \\ 0 & e^{p^*T} \end{bmatrix}, \quad p = -\sigma - j\omega$$

$$\Gamma = \int_0^T e^{Ft} G \, dt = \int_0^T \begin{bmatrix} e^{pt} \\ e^{p^*t} \end{bmatrix} dt = \begin{bmatrix} \frac{e^{pT}-1}{p} \\ \frac{e^{p^*T}-1}{p^*} \end{bmatrix}, \quad p = -\sigma - j\omega$$

$$\left(\int_0^T e^{at} dt = \frac{e^{at}}{a}\Big|_0^T = \frac{e^{aT}-1}{a} \right)$$

$H = [1 \;\; 1]$ (IGUAL AO DO SISTEMA CONTÍNUO)

b) $$\mathcal{C} = [\cancel{\Phi} \; \Gamma \;\; \Phi\Gamma] = \begin{bmatrix} \frac{e^{pT}-1}{p} & \frac{e^{pT}(e^{pT}-1)}{p} \\ \frac{e^{p^*T}-1}{p^*} & \frac{e^{p^*T}(e^{p^*T}-1)}{p^*} \end{bmatrix}$$

SISTEMA NÃO-CONTROLÁVEL.

$$\det \mathcal{C} = \frac{\cancel{(e^{pT}-1)} e^{p^*T} \cancel{(e^{p^*T}-1)}}{\cancel{pp^*}} - \frac{\cancel{(e^{p^*T}-1)} e^{pT} \cancel{(e^{pT}-1)}}{\cancel{pp^*}} = 0$$

SISTEMA NÃO-CONTROLÁVEL, SE: $e^{p^*T} = e^{pT} \iff \cancel{e^{-\sigma T}}(\cos wT + j\sin wT) = \cancel{e^{-\sigma T}}(\cos wT - j\sin wT)$

$j\sin wT = -j\sin wT \longrightarrow \sin wT = 0$

ENTÃO, $wT = k\pi$, PARA $k \in \mathbb{Z}^{+*}$.

$\boxed{f_s = \frac{w}{k\pi}}$, ONDE k É UM NÚMERO INTEIRO E ESTRITAMENTE POSITIVO.

FREQ. DE AMOSTRAGEM MÍNIMA: $f_{s,min} = \frac{w}{\pi} = \frac{2\pi f}{\pi} = 2f$ $\quad$ OBS.: $\forall f_s > f_{s,min}$, TEM-SE $\det \mathcal{C} \neq 0$.

c) $\mathcal{O} = \begin{bmatrix} H \\ H\Phi \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ e^{pT} & e^{p^*T} \end{bmatrix}$

$\det \mathcal{O} = 0$ (SISTEMA NÃO-OBSERVÁVEL) $\Rightarrow e^{pT} = e^{p^*T}$. PORTANTO, A RESPOSTA COM RELAÇÃO À OBSERVABILIDADE É A MESMA DO ITEM (b).

3 a) $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} [k_1 \ k_2] = \begin{bmatrix} 2-\frac{k_1}{2} & 1-\frac{k_2}{2} \\ -1 & 0 \end{bmatrix}$

$|zI - \Phi + \Gamma K| = \begin{vmatrix} z+\frac{k_1}{2}-2 & \frac{k_2}{2}-1 \\ 1 & z \end{vmatrix} = z^2 + \left(\frac{k_1}{2}-2\right)z + 1 - \frac{k_2}{2}$

$k_1 = 2$

$\alpha_c(z) = (z - \frac{1}{2} + \frac{1}{2}j)(z - \frac{1}{2} - \frac{1}{2}j) = z^2 - z + 1/2$ $\quad k_2 = 1$ $\quad K = [2 \ 1]$ //

b) $z = e^{sT} = e^{\sigma T} e^{jwT} = 0.5 + 0.5j = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right)$

$e^{\sigma T}(\cos jwT + j\sin wT) \qquad e^{\sigma T} \qquad \cos wT$

$e^{\sigma T} = \sqrt{2}/2 \longrightarrow 0.1\sigma = \ln(\sqrt{2}/2) \longrightarrow \sigma = -3.47$

$wT = \pi/4 \longrightarrow w = 10\pi/4 \longrightarrow w = 7.85$ $\quad s = -3.47 \pm 7.85j$ //

c) $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} [0 \ -1] = \begin{bmatrix} 2 & 1+\ell_1 \\ -1 & \ell_2 \end{bmatrix}$

$|zI - \Phi + LH| = \begin{vmatrix} z-2 & -1-\ell_1 \\ 1 & z-\ell_2 \end{vmatrix} = z^2 + (-\ell_2 - 2)z + 2\ell_2 + \ell_1 + 1$

$L_p = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ //

$\alpha_e(z) = z^2 \longrightarrow \ell_2 = -2 \longrightarrow -4 + \ell_1 + 1 = 0 \longrightarrow \ell_1 = 3$

$\Phi^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$; $L_a = \Phi^{-1} L_p = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} \longrightarrow L_a = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ //

d) Matriz Φ do compensador digital, baseado em estimador avançado:

$$\underbrace{\Phi - LH\Phi} - \Gamma K + LH\Gamma K = \longrightarrow \begin{bmatrix} -1 & 1/2 \\ 0 & 0 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix} [0 \ \ -1] \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}}_{\begin{bmatrix} 2 \\ -1 \end{bmatrix} [1 \ \ 0] = \begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$LH\Gamma K - \Gamma K = \underbrace{\begin{bmatrix} 2 \\ -1 \end{bmatrix} [0 \ \ -1] \overbrace{\begin{bmatrix} 1/2 \\ 0 \end{bmatrix} [2 \ \ 1]}^{\begin{bmatrix} 1 & 1/2 \\ 0 & 0 \end{bmatrix}}}_{[0 \ \ 0]} - \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} [2 \ \ 1] = \begin{bmatrix} -1 & -1/2 \\ 0 & 0 \end{bmatrix}$$

$$\hat{x}(k+1) = \begin{bmatrix} -1 & 1/2 \\ 0 & 0 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} y(k+1)$$

$$u(k) = -[2 \ \ 1]\, \hat{x}(k)$$

e) $$D(z) = -[2 \ \ 1] \begin{bmatrix} z+1 & -1/2 \\ 0 & z \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ -1 \end{bmatrix} z = \frac{-[2 \ \ 1] \begin{bmatrix} z & 1/2 \\ 0 & z+1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} z}{z^2 + z} = \frac{-(3z-2)\cancel{z}}{\cancel{z}(z+1)}$$

$$D(z) = \frac{-3z+2}{z+1}$$

f) $$G(z) = (0 \ \ -1) \begin{pmatrix} z-2 & -1 \\ 1 & z \end{pmatrix}^{-1} \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} = \frac{(0 \ \ -1) \begin{pmatrix} z & 1 \\ -1 & z-2 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}}{z^2 - 2z + 1} = \frac{1/2}{z^2 - 2z + 1}$$

$$\frac{Y(z)}{R(z)} = \frac{\frac{n_G}{d_G}}{1 - \frac{n_D}{d_D} \frac{n_G}{d_G}} = \frac{n_G d_D}{d_D d_G - n_D n_G}$$

Pólos: $d_D d_G - n_D n_G = (z+1)(z^2 - 2z + 1) - \frac{1}{2}(-3z+2) = 0$

$$z^3 - z^2 - z + \cancel{1} + \frac{3z}{2} - \cancel{1} = 0$$

$$z \underbrace{\left(z^2 - z + \frac{1}{2} \right)} = 0$$

$\alpha_e(z)$ dead-beat   ↪ $\alpha_c(s)$ (pólos $0.5 \pm 0.5j$) (ok).

4 a) $$\frac{Y(s)}{R(s)} = \frac{\frac{1}{s}}{1 + \frac{1}{s}\frac{(s+4)}{(s+3)}} = \frac{s+3}{s^2 + 3s + s + 4} = \frac{s+3}{(s+2)^2}$$

Zero: -3

Pólos (pólo duplo): -2 e -2

b) Zero: $e^{-0.9} = 0.41$ Pólo duplo: $e^{-0.6} = 0.55$

c) $\frac{G(s)}{s} = \frac{1}{s^2} \longrightarrow y(t) = kTu(t) \longrightarrow y(k) = T.ku(k) \longrightarrow Y(z) = \frac{T.z}{(z-1)^2}$

$$G(z) = \left(\frac{z-1}{z} \right) \frac{Tz}{(z-1)^2} \longrightarrow \boxed{G(z) = \frac{T}{(z-1)}}$$

$$\left(T = 0.3 \Rightarrow G(z) = \frac{0.3}{z-1} \right)$$

d) $D(z) = \dfrac{-\left(\frac{2}{T}\frac{(z-1)}{(z+1)} + 4\right)}{\left(\frac{2}{T}\frac{(z-1)}{(z+1)} + 3\right)} = \dfrac{-\left((2+4T)z + 4T-2\right)}{(2+3T)z + 3T-2}$

$T = 0.3 \longrightarrow D(z) = \dfrac{-(3.2z-0.8)}{2.9z-1.1}$

$\dfrac{Y(z)}{R(z)} = \dfrac{\frac{0.3}{z-1}}{1 + \frac{0.3}{z-1}\frac{(3.2z-0.8)}{(2.9z-1.1)}} = \dfrac{0.3(2.9z-1.1)}{(2.9z^2-4z+1.1)+0.96z-0.24} = \dfrac{0.3(2.9z-1.1)}{2.9z^2-3.04z+0.86}$

ZERO: 0.38 PÓLOS: $0.52 \pm 0.15j$

e) $D(z) = -(z+a)/(z+b)$

$\dfrac{Y(z)}{R(z)} = \dfrac{\frac{0.3}{z-1}}{1 + \frac{0.3}{z-1}\frac{(z+a)}{(z+b)}} = \dfrac{0.3(z+b)}{z^2+(b-1)z-b+0.3z+0.3a} = \dfrac{0.3(z+b)}{z^2+(b-0.7)z+0.3a-b}$

f) $\alpha_c(z) = (z-0.55)^2 = z^2 - 1.1z + 0.3$ (0.3025)

COMPARANDO COM O DENOMINADOR DO ITEM (e): $b - 0.7 = -1.1 \longrightarrow b = -0.4$

$0.3a - b = 0.3 \longrightarrow 0.3a = -0.1 \longrightarrow a = -\frac{1}{3}$

USANDO (a) E (b) CALCULADOS: ZERO: 0.4

PÓLOS: $z^2 - 1.1z + 0.3 = 0 \longrightarrow z = 0.5$ E $z = 0.6$

(SÃO APROXIMADAMENTE IGUAIS A 0.55. A DIFERENÇA VEM DO ARREDONDAMENTO DE 0.3025 PARA 0.3).

g) $T = 0.5$: ALTERAÇÃO NO ITEM (d):

$D(z) = \dfrac{-4z}{3.5z-0.5}$

$\dfrac{Y(z)}{R(z)} = \dfrac{\frac{0.5}{z-1}}{1 + \frac{0.5}{z-1}\cdot\frac{4z}{(3.5z-0.5)}} = \dfrac{0.5(3.5z-0.5)}{(z-1)(3.5z-0.5)+2z} = \dfrac{0.5(3.5z-0.5)}{3.5z^2-2z+0.5}$

ZERO: 0.14 PÓLOS: $0.29 \pm 0.25j$

$T = 0.5$: ALTERAÇÃO NO ITEM (f):

$e^{-1.5} = 0.22$ (ZERO) E $e^{-1.0} = 0.37$ (PÓLO DUPLO)

$\alpha_c(z) = (z-0.37)^2 = z^2 - 0.74z + 0.14$ (T ALTERADO)

COMPARANDO COM O DENOMINADOR DO ITEM (e): $b - 0.5 = -0.74 \longrightarrow b = -0.24$

$0.5a - b = 0.14 \longrightarrow a = -0.2$

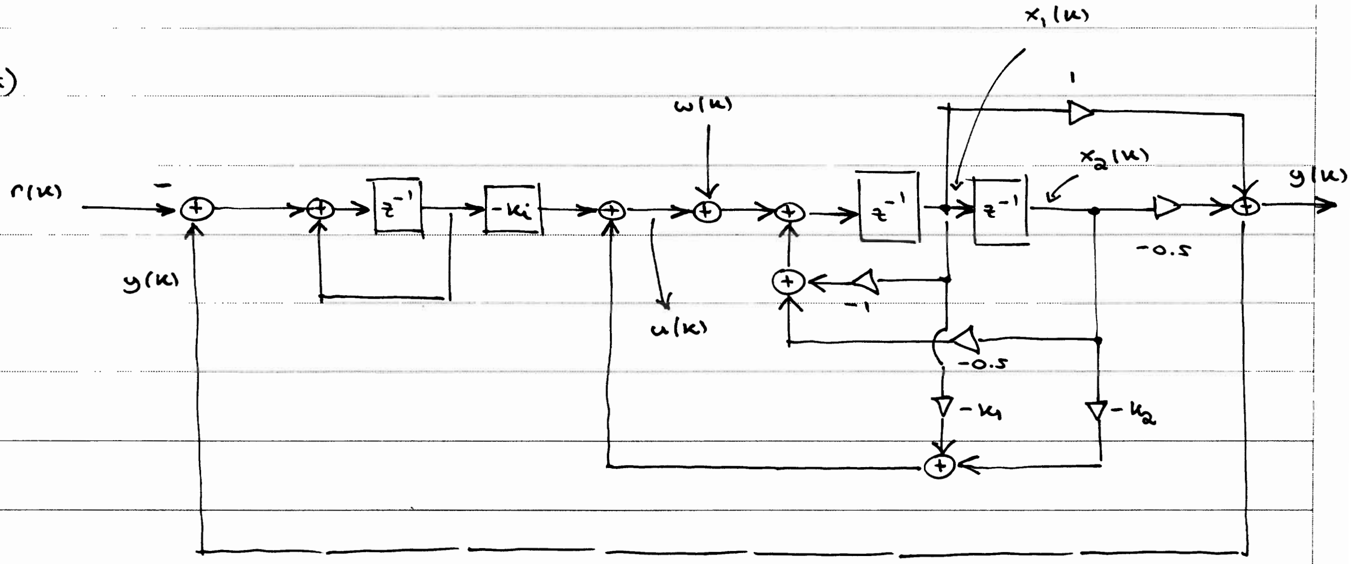
USANDO (a) E (b) CALCULADOS: ZERO: 0.24

PÓLOS: $z^2 + (b-0.5z) + 0.5a - b = z^2 - 0.74z + 0.14 = 0$

$0.37 \pm 0.06j$ PARTE COMPLEXA PEQUENA, CAUSADA POR ARREDONDAMENTO.

OBS.: NOTE QUE, PARA $T = 0.5$, OCORRE MAIOR AFASTAMENTO ENTRE OS PÓLOS E ZEROS DO SISTEMA COM COMPENSADOR APROXIMADO $(0.14$ E $0.29 \pm 0.25j)$ E OS PÓLOS E ZEROS DO MAPEAMENTO $e^{0.5s}$ $(0.22$ E $0.37)$. NO CASO DO COMPENSADOR EXATO, O ERRO É BEM MENOR $(0.24$ E $0.37 \pm 0.06j)$.

5) a)



b) FCC: $x(k+1) = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(k)$

$y(k) = [1 \quad -0.5]\, x(k)$

$$\begin{bmatrix} x_i(k+1) \\ x(k) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & H \\ -\Gamma k_i & \Phi - \Gamma k \end{bmatrix}}_{\Phi_i} \begin{bmatrix} x_i(k) \\ x(k) \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(k) + \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} w(k)$$

$$\Phi_i = \begin{bmatrix} 1 & 1 & -0.5 \\ -k_i & -1-k_1 & -0.5-k_2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$|zI - \Phi_i| = \begin{vmatrix} z-1 & -1 & 0.5 \\ k_i & z+k_1+1 & k_2+0.5 \\ 0 & -1 & z \end{vmatrix} = z\left(z^2 + (k_1+1-1)z - k_1 - 1 + k_i\right)$$

$$+ 1\left((k_2+0.5)z - k_2 - 0.5 - 0.5k_i\right)$$

$$= z^3 + k_1 z^2 + \left(k_i - k_1 + k_2 - \tfrac{1}{2}\right)z + \left(-0.5k_i - k_2 - 0.5\right)$$

$\alpha_c(z) = z^3 \longrightarrow k_1 = 0$

E $k_i + k_2 = 1/2$

$-0.5k_i - k_2 = 1/2$

$0.5k_i = 1 \longrightarrow k_i = 2$

$k_2 = -3/2$

$[k_i \quad k_1 \quad k_2] = [2 \quad 0 \quad -1.5]$