CONTROLE LINEAR II —— PROVA PARCIAL # 2 —— GABARITO

QUESTÃO # 1

a) $$D(z) = D(s)\Big|_{s=\frac{z-1}{Tz}} = \frac{\frac{z-1}{Tz}+1}{\left(\frac{z-1}{Tz}+2\right)\left(\frac{z-1}{Tz}+3\right)} = \frac{(T^2+1)z^2 - Tz}{((1+2T)z-1)((1+3T)z-1)}$$

$$= \frac{(T^2+1)z^2 - Tz}{(6T^2+5T+1)z^2-(5T+2)z+1} \overset{T=0.1}{=} \frac{1.01z^2-0.1z}{1.56z^2-2.5z+1.0}$$

Errata (22/06/2005): o coeficiente do termo z^2, no numerador, nao e' T^2+1 = 1.01, e sim T^2+T = 0.11.

PÓLOS : $z = \frac{2.5 \pm \sqrt{2.5^2-4\times 1\times 1.56}}{2\times 1.56} = \frac{2.5 \pm 0.1}{2\times 1.56}$ — 0.83, 0.77

ZEROS : 0 E 0.099

b) $$D(z) = D(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} = \frac{\frac{2}{T}\frac{z-1}{z+1}+1}{\left(\frac{2}{T}\frac{z-1}{z+1}+2\right)\left(\frac{2}{T}\frac{z-1}{z+1}+3\right)} = \frac{(T^2+2T)z^2+2T^2z+T^2-2T}{((2T+2)z+2T-2)((3T+2)z+3T-2)}$$

$$= \frac{(T^2+2T)z^2+2T^2z+T^2-2T}{(6T^2+10T+4)z^2+(12T^2-8)z+6T^2-10T+4} \overset{T=0.1}{=} \frac{0.21z^2+0.02z-0.19}{5.06z^2-7.88z+3.06}$$

PÓLOS : $z = \frac{7.88 \pm \sqrt{7.88^2-4\times 3.06\times 5.06}}{2\times 5.06} = \frac{7.88\pm 0.4}{10.12}$ — 0.82, 0.74

ZEROS : $z = \frac{-0.02 \pm \sqrt{0.02^2+4\times 0.19\times 0.21}}{0.42} = \frac{-0.02\pm 0.4}{0.42}$ — 0.9, -1.0

c) $$D(s) = \frac{A}{s+2}+\frac{B}{s+3} = \frac{-1}{s+2}+\frac{2}{s+3}$$

$$F = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \qquad G = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad H = \begin{bmatrix} -1 & 2 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} e^{-2T} & 0 \\ 0 & e^{-3T} \end{bmatrix} \qquad \Gamma = \int_0^T \begin{bmatrix} e^{-2\tau} \\ e^{-3\tau} \end{bmatrix} d\tau = \begin{bmatrix} \frac{e^{-2\tau}}{-2} \\ \frac{e^{-3\tau}}{-3} \end{bmatrix}\Bigg|_0^T = \begin{bmatrix} \frac{1}{2}(1-e^{-2T}) \\ \frac{1}{3}(1-e^{-3T}) \end{bmatrix}$$

$$D(z) = \frac{\begin{bmatrix} -1 & 2 \end{bmatrix} \begin{bmatrix} z-e^{-3T} & 0 \\ 0 & z-e^{-2T} \end{bmatrix} \begin{bmatrix} \frac{1}{2}(1-e^{-2T}) \\ \frac{1}{3}(1-e^{-3T}) \end{bmatrix}}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}}$$

$$= \frac{\left(\frac{2}{3}(1-e^{-3T})-\frac{1}{2}(1-e^{-2T})\right)z + \left(\frac{1}{2}e^{-3T}(1-e^{-2T})-\frac{2}{3}e^{-2T}(1-e^{-3T})\right)}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}} \overset{T=0.1}{=} \frac{0.082z-0.074}{z^2-1.56z+\cancel{0.61}\ 0.6068}$$

PÓLOS : 0.82 E 0.74 $(e^{-0.2}$ E $e^{-0.3})$

ZERO : 0.902

QUESTÃO #2:

a) $G(s) = \dfrac{0.1s + 2}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2} = \dfrac{1.9}{s+1} - \dfrac{1.8}{s+2}$

$F = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \quad G = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad H = \begin{bmatrix} 1.9 & -1.8 \end{bmatrix}$

$\Phi_0 = \begin{bmatrix} e^{-T} & 0 \\ 0 & e^{-2T} \end{bmatrix} \underset{T=0.01}{=} \begin{bmatrix} 0.99 & 0 \\ 0 & 0.98 \end{bmatrix} \qquad \Gamma_0 = \begin{bmatrix} (1-e^{-T}) \\ \frac{1}{2}(1-e^{-2T}) \end{bmatrix} \underset{T=0.01}{=} \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}$

PARA USAR A FORMA CANÔNICA CONTROLÁVEL:

$G(z) = \dfrac{[1.9 \quad -1.8] \begin{bmatrix} z-0.98 & 0 \\ 0 & z-0.99 \end{bmatrix} \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}}{(z-0.98)(z-0.99)} = \dfrac{0.01\,(0.1z - 0.08)}{z^2 - 1.97z + 0.9702} = \dfrac{10^{-3}(z-0.8)}{z^2 - 1.97z + 0.97}$

A PARTIR DA EXPRESSÃO DE G(z):

$\Phi = \begin{bmatrix} 1.97 & -0.97 \\ 1 & 0 \end{bmatrix} \quad \Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad H = 10^{-3}\begin{bmatrix} 1 & -0.8 \end{bmatrix}$

b) $z_1, z_2 = e^{-0.1 \pm 0.1j} = 0.9 \pm 0.1j$

$\alpha_c(z) = (z - 0.9 - 0.1j)(z - 0.9 + 0.1j) = z^2 - 1.8z + 0.82$

$\Phi - \Gamma K = \begin{bmatrix} 1.97 - K_1 & -0.97 - K_2 \\ 1 & 0 \end{bmatrix}$

$|zI - \Phi + \Gamma K| = \begin{vmatrix} z + K_1 - 1.97 & K_2 + 0.97 \\ -1 & z \end{vmatrix} = z^2 + (K_1 - 1.97)z + K_2 + 0.97$

$K_1 = 0.17 \quad K_2 = -0.15 \longrightarrow K = [0.17 \quad -0.15]$

c) $x(k+1) = \begin{bmatrix} 1.8 & -0.82 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(k)$

$y(k) = 10^{-3}\begin{bmatrix} 1 & -0.8 \end{bmatrix} x(k)$

d) $\dfrac{Y(z)}{R(z)} = 10^{-3}\begin{bmatrix} 1 & -0.8 \end{bmatrix} \begin{bmatrix} z-1.8 & 0.82 \\ -1 & z \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \dfrac{10^{-3}\begin{bmatrix} 1 & -0.8 \end{bmatrix} \begin{bmatrix} z & -0.82 \\ 1 & z-1.8 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{z^2 - 1.8z + 0.82}$

$\dfrac{Y(z)}{R(z)} = \dfrac{10^{-3}(z-0.8)}{z^2 - 1.8z + 0.82} \longrightarrow$ OS PÓLOS ESTÃO NAS POSIÇÕES CORRETAS (NOTE QUE O DENOMINADOR É $\alpha_c(z)$).

e) $\lim\limits_{z \to 1} \dfrac{Y(z)}{R(z)} = \dfrac{2\times10^{-4}}{2\times10^{-2}} = 0.01 \longrightarrow \bar{N} = 100$

QUESTÃO #3:

a) $\alpha_e(z) = (z+0.5+0.5j)(z+0.5-0.5j) = z^2 + z + 0.5$

$$\Phi - LH = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} -\ell_1 & 1 \\ 1-\ell_2 & 0 \end{bmatrix}$$

$$\begin{vmatrix} z+\ell_1 & -1 \\ \ell_2 - 1 & z \end{vmatrix} = z^2 + \ell_1 z + \ell_2 - 1 \longrightarrow L = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$$

b) $$\Phi - LH\Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1-\ell_1 \\ 1 & -\ell_2 \end{bmatrix}$$

$$\begin{vmatrix} z & \ell_1 - 1 \\ -1 & z+\ell_2 \end{vmatrix} = z^2 + \ell_2 z + \ell_1 - 1 \longrightarrow L = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$$

ALTERNATIVA: $$\underbrace{\begin{bmatrix} 1.5 \\ 1 \end{bmatrix}}_{L_{ATUAL}} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\Phi^{-1}} \underbrace{\begin{bmatrix} 1 \\ 1.5 \end{bmatrix}}_{L_{PREDIÇÃO}}$$

c) $$\Phi - LH\Phi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1.5 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -0.5 \\ 1 & -1 \end{bmatrix}$$

$$\Gamma - LH\Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1.5 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.5 \\ -1 \end{bmatrix}$$

$K = [0 \quad 1]$

$$-(\Gamma - LH\Gamma)K = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0.5 \\ 0 & 1 \end{bmatrix}$$

$$\Phi - LH\Phi - \Gamma K + LH\Gamma K = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\hat{x}(k+1) = (\Phi - LH\Phi)\hat{x}(k) + (\Gamma - LH\Gamma)\underbrace{u(k)}_{-K\hat{x}(k)} + Ly(k+1)$$

ENTÃO: $$\hat{x}(k+1) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1.5 \\ 1 \end{bmatrix} y(k+1)$$

$$u(k) = [0 \quad 1]\,\hat{x}(k)$$

$$\frac{U(z)}{Y(z)} = [0 \quad 1] \begin{bmatrix} z & 0 \\ -1 & z \end{bmatrix}^{-1} \begin{bmatrix} 1.5 \\ 1 \end{bmatrix} \underbrace{z}_{\text{ATENÇÃO}} = \frac{[0 \quad 1] \begin{bmatrix} z & 0 \\ 1 & z \end{bmatrix} \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}}{z^2} z = \frac{z+1.5}{z}$$

$$\frac{U(z)}{Y(z)} = \frac{z+1.5}{z}$$

QUESTÃO #4:

a) $z = Px = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x \qquad T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$Tz = \Phi T z + \Gamma u$

$y = HTz$

$$\Phi_z = \underbrace{\overset{T^{-1}}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}\overset{\Phi}{\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}}}_{\begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}}\overset{T}{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \quad ; \quad \Gamma_z = \overset{T^{-1}}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}\overset{\Gamma}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H_z = \underset{H}{\begin{bmatrix} 1 & 1 \end{bmatrix}}\underset{T}{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

ENTÃO: $z(k+1) = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} z(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$

$y(k) = [1 \quad 0]\, z(k)$

$\alpha_e(z) = z + 0.2$

$\Phi_{bb} - L\Phi_{ab} = -1 + 4L$

$zI - \Phi_{bb} + L\Phi_{ab} = z + 1 - 4L = z + 0.2 \longrightarrow L = 0.2$

b) $u = -K\hat{x} = -\underbrace{KT}_{=K_z}\hat{z} \qquad K_z = [1 \quad -0.5]\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = [1 \quad -1.5]$

$u(k) = -K_{z1}\, y(k) - K_{z2}\, \hat{z}_2(k)$

$z_1(k) = y(k)$

$\hat{z}_2(k+1) = \Phi_{bb}\hat{z}_2(k) + \Phi_{ba} z_1(k) + \Gamma_b u(k) + L z_1(k+1) - L\Phi_{aa} z_1(k) - L\Gamma_a u(k) - L\Phi_{ab}\hat{z}_2(k)$

$\hat{z}_2(k+1) = (\Phi_{bb} - L\Phi_{ab})\hat{z}_2(k) + (\Phi_{ba} - L\Phi_{aa}) y(k) + L y(k+1) + (\Gamma_b - L\Gamma_a) u(k)$

$$\hat{z}_2(k+1) = \underbrace{\Big(\underbrace{\Phi_{bb} - L\Phi_{ab}}_{-0.2} - K_{z2}(\underbrace{\Gamma_b - L\Gamma_a}_{-0.2})\Big)}_{-0.5}\hat{z}_2(k) + \underbrace{\Big(\underbrace{\Phi_{ba} - L\Phi_{aa}}_{0.4} - K_{z1}(\underbrace{\Gamma_b - L\Gamma_a}_{-0.2})\Big)}_{0.6} y(k) + L y(k+1)$$

ENTÃO: $\hat{z}_2(k+1) = -0.5\,\hat{z}_2(k) + 0.6\, y(k) + 0.2\, y(k+1)$

c) $\hat{z}_2(z) = \dfrac{(0.2z + 0.6)}{z + 0.5} Y(z) \qquad$ (PORQUE $u(k) = -z_1(k) + 1.5\,\hat{z}_2(k)$)

$U(z) = -Y(z) + 1.5\,\hat{z}_2(z) = Y(z)\left(-1 + \dfrac{0.3z + 0.9}{z + 0.5}\right) \longrightarrow \dfrac{U(z)}{Y(z)} = \dfrac{-0.7z + 0.4}{z + 0.5}$

d) $G(z) = \dfrac{z+1}{z^2 - 2z + 1}$

$$\frac{Y(z)}{R(z)} = \frac{G(z)}{1 - D(z)G(z)} = \frac{(z+1)(z+0.5)}{(z^2 - 2z + 1)(z + 0.5) - (z+1)(-0.7z + 0.4)}$$

$$\frac{Y(z)}{R(z)} = \frac{z^2 + 1.5z + 0.5}{z^3 - 0.8z^2 + 0.3z + 0.1} \longleftarrow \text{DENOMINADOR IGUAL A } \underbrace{(z^2 - z + 0.5)}_{\alpha_c(z)}\underbrace{(z + 0.2)}_{\alpha_e(z)} \quad \text{(OK)}$$

QUESTÃO #5:

a) $$\begin{bmatrix} x_i(k+1) \\ x(k+1)\end{bmatrix} = \underbrace{\begin{bmatrix}1&1\\0&0.4\end{bmatrix}}_{\Phi_i}\begin{bmatrix}x_i(k)\\x(k)\end{bmatrix} + \underbrace{\begin{bmatrix}0\\2\end{bmatrix}}_{\Gamma_i} u(k) - \begin{bmatrix}1\\0\end{bmatrix} r(k) + \begin{bmatrix}0\\2\end{bmatrix} w(k)$$

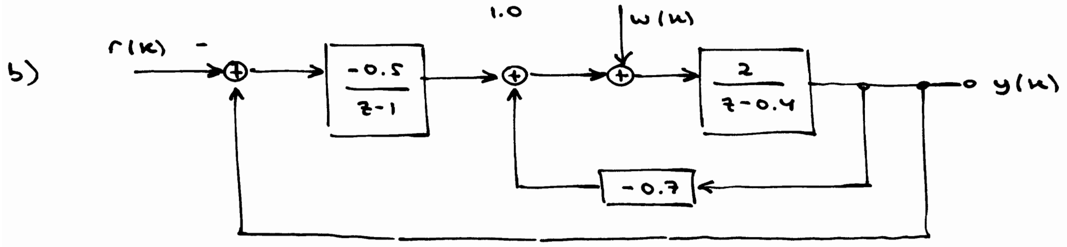
$$[\Phi_i - \Gamma_i K] = \begin{bmatrix}1&1\\0&0.4\end{bmatrix} - \begin{bmatrix}0\\2\end{bmatrix}[k_1 \quad k_2] = \begin{bmatrix}1&1\\-2k_1&0.4-2k_2\end{bmatrix}$$

$$|zI - \Phi_i + \Gamma_i K| = \begin{vmatrix} z-1 & -1 \\ 2k_1 & z+2k_2-0.4\end{vmatrix} = z^2 + (2k_2 - 1.4)z + 0.4 - 2k_2 + 2k_1$$

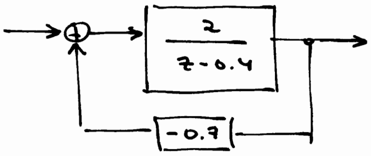
$$\alpha_c(z) = z^2 \longrightarrow k_2 = 0.7$$

$$2k_1 = \underbrace{2k_2 - 0.4}_{1.0} \longrightarrow k_1 = 0.5 \quad \Longrightarrow \quad K = [\underbrace{0.5}_{K_i} \;|\; \underbrace{0.7}_{K_o}]$$

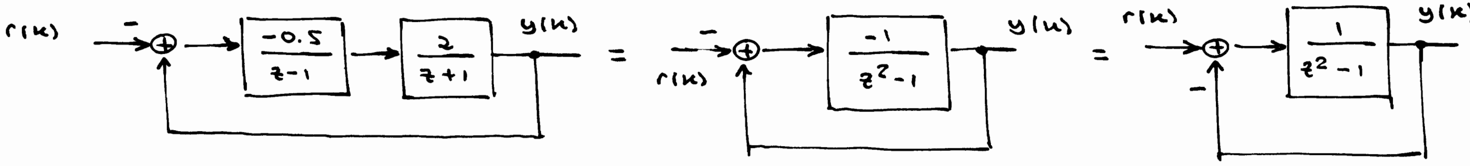
b)



w(k) = 0:



$$= \frac{\frac{2}{z-0.4}}{1 + \frac{2\times 0.7}{z-0.4}} = \frac{2}{z+1}$$



ENTÃO: $$\frac{Y(z)}{R(z)} = \frac{\frac{1}{z^2-1}}{1 + \frac{1}{z^2-1}} = \frac{1}{z^2}$$ NOTE QUE $\lim_{z \to 1} \frac{Y(z)}{R(z)} = 1.0$

c) $$\begin{bmatrix} x_i(k+1) \\ x(k+1)\end{bmatrix} = \begin{bmatrix}1&1\\0&0.4\end{bmatrix}\begin{bmatrix}x_i(k)\\x(k)\end{bmatrix} + \begin{bmatrix}0\\2\end{bmatrix} \underset{-0.5x_i(k) - 0.7x(k)}{\underset{\uparrow}{u(k)}} + \begin{bmatrix}0\\2\end{bmatrix} w(k) \longleftarrow (r(k) = 0)$$

$$\begin{bmatrix} x_i(k+1) \\ x(k+1)\end{bmatrix} = \begin{bmatrix}1&1\\-1&-1\end{bmatrix}\begin{bmatrix}x_i(k)\\x(k)\end{bmatrix} + \begin{bmatrix}0\\2\end{bmatrix} w(k)$$

NOTE QUE $\lim_{z \to 1} \frac{Y(z)}{W(z)} = 0.0$

$$y(k) = [0 \quad 1]\begin{bmatrix}x_i(k)\\x(k)\end{bmatrix}$$

$$\frac{Y(z)}{W(z)} = [0 \quad 1]\begin{bmatrix}z-1&-1\\1&z+1\end{bmatrix}^{-1}\begin{bmatrix}0\\2\end{bmatrix} = \frac{[0 \quad 1]\begin{bmatrix}z+1&1\\-1&z-1\end{bmatrix}\begin{bmatrix}0\\2\end{bmatrix}}{z^2-1+1} = \frac{2(z-1)}{z^2}$$