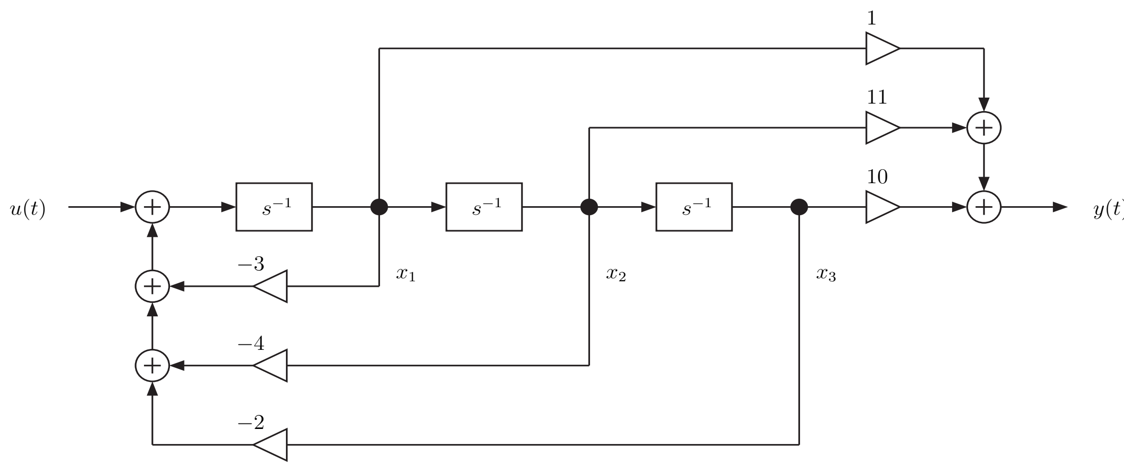
UFRJ / Escola Politécnica / DEL – Primeiro Semestre de 2009
EEL-760 – Controle Linear II
Prova Parcial #1 – 27 de abril de 2009 – Gabarito

QUESTÃO #1:

a) $G(s) = \dfrac{s^2+11s+10}{(s^2+2s+2)(s+1)} = \dfrac{s^2+11s+10}{s^3+3s^2+4s+2}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 11 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



b) $Y(s) = \dfrac{s+10}{s(s^2+2s+2)}$

$$\frac{1}{2}.\frac{2}{[(s+1)^2+1^2]} \longleftrightarrow \frac{1}{2}.2e^{-t}\sin(t)u(t) = e^{-t}\sin(t)u(t)$$

$$5.\frac{2}{s[(s+1)^2+1^2]} \longleftrightarrow 5[1-e^{-t}(\cos(t)+\sin(t))]u(t)$$

Então: $y(t) = [5 - 5e^{-t}\cos(t) - 4e^{-t}\sin(t)]u(t)$

c) $Y(s) = \begin{bmatrix} 1 & 11 & 10 \end{bmatrix} \begin{bmatrix} s+3 & 4 & 2 \\ -1 & s & 0 \\ 0 & -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$$Y(s) = \begin{bmatrix} 1 & 11 & 10 \end{bmatrix} \frac{\begin{bmatrix} s^2 & -4s-2 & -2s \\ s & s^2+3s & -2 \\ 1 & s+3 & s^2+3s+4 \end{bmatrix}}{s^3+3s^2+4s+2} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$Y(s) = \begin{bmatrix} s^2+11s+10 & 11s^2+39s+28 & 10s^2+28s+18 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} . \frac{1}{s^3+3s^2+4s+2} = 0 \rightarrow y(t) = 0$$

Se $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, então:

$$Y(s) = \begin{bmatrix} s^2+11s+10 & 11s^2+39s+28 & 10s^2+28s+18 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} . \frac{1}{s^3+3s^2+4s+2} = \frac{s+10}{s^2+2s+2}$$

$$y(t) = [5e^{-t}\cos(t) + 5e^{-t}\sin(t) + 4e^{-t}\sin(t) - 4e^{-t}\cos(t)]u(t) = [e^{-t}\cos(t) + 9e^{-t}\sin(t)]u(t)$$

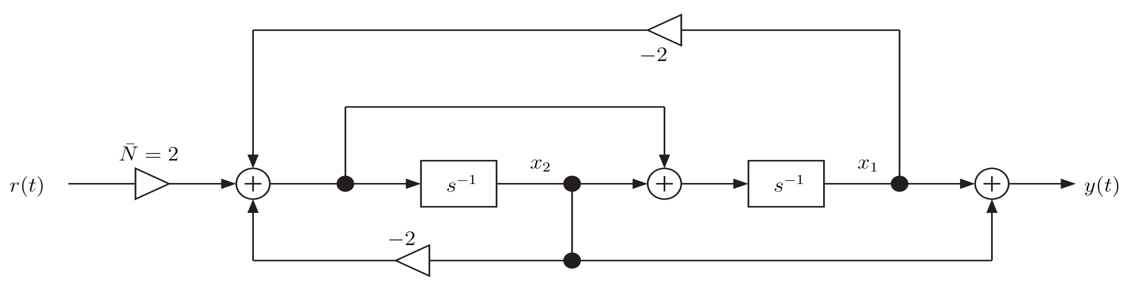
d) Desprezando o efeito do zero em $s = -10$, temos: $\sigma = 1$ seg^{-1}, $\omega_d = 1$ rad/seg, $\omega_n = \sqrt{2}$ rad/seg e $\xi = 0.7 \longrightarrow$ $t_s = 4.6$ seg, $t_p = \pi$ seg, $t_r = 1.27$ seg e $M_{\text{p}} = 5\%$.

QUESTÃO #2:

a)
$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ -2 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r$$
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{x}$$

$$\frac{Y(s)}{R(s)} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ 2 & s+2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \frac{\begin{bmatrix} s+2 & -1 \\ -2 & s+2 \end{bmatrix}}{s^2+4s+2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{2s+1}{s^2+4s+2}$$

b) $\lim_{s\to 0} \frac{Y(s)}{R(s)} = 1/2 \longrightarrow \bar{N} = 2$:



c) $\mathbf{P} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\qquad$ $\mathbf{T} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$\mathbf{F}_{\text{z}} = \mathbf{T}^{-1}\mathbf{F}\mathbf{T} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$F_{\text{bb}} - LF_{\text{ab}} = -L \longrightarrow s + L = s + 20 \longrightarrow L = 20$$

d) $Y(s) = \frac{2s+1}{s^2+4s+2} + 1 = \frac{s^2+6s+3}{s^2+4s+2}$

QUESTÃO #3:

a) $\mathbf{K} = \begin{bmatrix} 4 & 8 \end{bmatrix}$

b) $\mathbf{L} = \begin{bmatrix} 100 \\ 20 \end{bmatrix}$

c) $$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{F}\mathbf{x} - \mathbf{G}\mathbf{K}\hat{\mathbf{x}} \text{ (pois } \bar{N} = 0) \\ \dot{\hat{\mathbf{x}}} &= \mathbf{F}\hat{\mathbf{x}} - \mathbf{G}\mathbf{K}\hat{\mathbf{x}} + \mathbf{L}\mathbf{H}\mathbf{x} - \mathbf{L}\mathbf{H}\hat{\mathbf{x}} - \mathbf{L}r \text{ (pois } \mathbf{M} = -\mathbf{L}) \end{aligned}$$

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\hat{\mathbf{x}}} \end{bmatrix} = \begin{bmatrix} \mathbf{F} & -\mathbf{G}\mathbf{K} \\ \mathbf{L}\mathbf{H} & \mathbf{F} - \mathbf{G}\mathbf{K} - \mathbf{L}\mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ -\mathbf{L} \end{bmatrix} r$$

$$y = \begin{bmatrix} \mathbf{H} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \hat{\mathbf{x}} \end{bmatrix}$$

$$\mathbf{F}_c = \begin{bmatrix} 0 & 0 & -4 & -8 \\ 1 & 0 & 0 & 0 \\ 0 & 100 & -4 & -108 \\ 0 & 20 & 1 & -20 \end{bmatrix} \qquad \mathbf{G}_c = \begin{bmatrix} 0 \\ 0 \\ -100 \\ -20 \end{bmatrix} \qquad \mathbf{H}_c = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

d) $\mathbf{H}_c \left(s\mathbf{I} - \mathbf{F}_c \right)^{-1} \mathbf{G}_c = \dfrac{560s + 800}{s^4 + 24s^3 + 188s^2 + 560s + 800}$

$$\text{Obs.: } \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s & 0 & 4 & 8 \\ -1 & s & 0 & 0 \\ 0 & -100 & s+4 & 108 \\ 0 & -20 & -1 & s+20 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -100 \\ -20 \end{bmatrix} =$$

$$= \frac{1}{\alpha_c(s)\alpha_e(s)} \left[(+100) \begin{vmatrix} s & 4 & 8 \\ -1 & 0 & 0 \\ 0 & -1 & s+20 \end{vmatrix} + (-20) \begin{vmatrix} s & 4 & 8 \\ -1 & 0 & 0 \\ 0 & s+4 & 108 \end{vmatrix} \right] =$$

$$\frac{100(4s + 88) - 20(-8s + 400)}{\alpha_c(s)\alpha_e(s)}$$

Obs.: como alternativa ao método acima, pode-se obter $\gamma(s)$ através da relação $\gamma(s) = \begin{vmatrix} s\mathbf{I} - \mathbf{F} & \mathbf{L} \\ -\mathbf{K} & 0 \end{vmatrix}$

Por causa do zero em $s = -10/7 = -1.43$, o tempo de subida será menor do que 0.64 seg (que seria obtido com $\omega_n = 2\sqrt{2}$) e o *overshoot* será maior do que 5% (que seria obtido com $\omega_d = \sigma$). Uma simulação indicaria t_r em torno de 0.31 seg e *overshoot* em torno de 36%.

QUESTÃO #4:

a) $x_{10}^3 = 0 \rightarrow x_{10} = 0 \qquad \ln x_{20} = 2 \rightarrow x_{20} = e^2 \qquad e^{x_{30}} = 2 \rightarrow x_{30} = \ln 2$

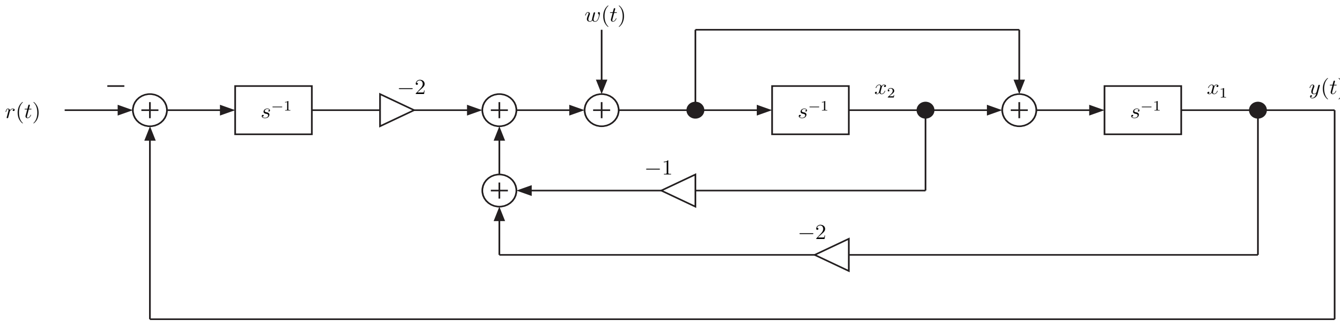
b) $\mathbf{F} = \begin{bmatrix} 3x_{10}^2 & 0 & 0 \\ 0 & 1/x_{20} & 0 \\ 0 & 0 & e^{x_{30}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & e^{-2} & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

c) $x_{10} - 1 = -1 \rightarrow x_{10} = 0 \qquad x_{20} - 1 = e \rightarrow x_{20} = e + 1 \qquad x_{30} - 1 = 0 \rightarrow x_{30} = 1$

d) $\mathbf{F} = \begin{bmatrix} 3(x_{10} + u_0)^2 & 0 & 0 \\ 0 & 1/(x_{20} + u_0) & 0 \\ 0 & 0 & e^{(x_{30} + u_0)} \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & e^{-1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 3(x_{10} + u_0)^2 \\ 1/(x_{20} + u_0) \\ e^{(x_{30} + u_0)} \end{bmatrix} = \begin{bmatrix} 3 \\ e^{-1} \\ 1 \end{bmatrix}$

QUESTÃO #5:

a) Diagrama de blocos de controle integral:



b) $$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s & \boxed{-1} & 0 \\ 2 & s+2 & 0 \\ 2 & 2 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \frac{2(s+1)}{s^3+3s^2+4s+2} = \frac{2}{s^2+2s+2}$$

$$Y(s) = \frac{2}{s(s^2+2s+2)} \longrightarrow y(t) = [1 - e^{-t}(\cos(t) + \sin(t))]u(t)$$

c) $$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 & 0 \\ 2 & \boxed{s+2} & 0 \\ 2 & \boxed{2} & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \frac{s(s+1)}{s^3+3s^2+4s+2} = \frac{s}{s^2+2s+2}$$

$$Y(s) = \frac{1}{s^2+2s+2} \longrightarrow y(t) = e^{-t}\sin(t)u(t)$$

d) $$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 & 0 \\ 2 & s+2 & 0 \\ 2 & \boxed{2} & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \longrightarrow y(t) = 0$$