Universidade Federal do Rio de Janeiro

Escola Politécnica

DATA: 26/9/2008

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Aluno: GABARITO — P1 EEL760 2008/2

Disciplina: CONTROLE LINEAR II - A

Turma: EEL760

Professor: JOSÉ GABRIEL R.C. GOMES

QUESTÃO #1:

a) $F = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 1 \\ b \end{bmatrix}$ $\quad H = [1 \; 0]$

$\mathcal{C} = \begin{bmatrix} 1 & b-3 \\ b & -2 \end{bmatrix}$ $\quad -b^2+3b-2=0$ $\quad b = \frac{3 \pm \sqrt{1}}{2}$ $\boxed{b=1 \text{ ou } b=2}$

$b^2-3b+2=0$

b) $Y(s) = \frac{1}{s}[1 \; 0]\begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ \cancel{b}\,3 \end{bmatrix} = [1 \; 0]\frac{\begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}}{s^2+3s+2}\begin{bmatrix} 1 \\ 3 \end{bmatrix}\cdot\frac{1}{s}$

$Y(s) = \frac{s+3}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$

$A = 3/2 \quad B = 2/(-1) = -2 \quad C = 1/2$

$\boxed{y(t) = \left(\frac{3}{2} - 2e^{-t} + \frac{1}{2}e^{-2t}\right)u(t)}$

c) $Y(s) = [1 \; 0]\frac{\begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}}{s^2+3s+2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{s+1}{(s+1)(s+2)} = \frac{1}{s+2}$; $\boxed{y(t) = e^{-2t}u(t)}$

d) $Y(s) = [1 \; 0]\frac{\begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}}{s^2+3s+2}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{s-1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$

$A = -2 \quad B = 3$

$\boxed{y(t) = \left(-2e^{-t} + 3e^{-2t}\right)u(t)}$

QUESTÃO #2:

a) $G(s) = [21 \; -19]\begin{bmatrix} s+16 & -13 \\ 17 & s-12 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\cdot\frac{1}{2} = \frac{[21 \; -19]}{2}\frac{\begin{bmatrix} s-12 & 13 \\ -17 & s+16 \end{bmatrix}}{s^2+4s+29}$

$F = \begin{bmatrix} -16 & 13 \\ -17 & 12 \end{bmatrix}$

$+21\times13$

$$G(s) = \frac{1}{2}\cdot\frac{(21s-19s+21\times(-12)+19\times\cancel{17}+(-19)\times16)}{s^2+4s+29} = \frac{s+20}{s^2+4s+29}$$

PÓLOS: $s=-2\pm5j$ ZERO: $s=-20$ (EFEITO DESPREZÍVEL SOBRE A RESPOSTA DO DEGRAU).

$\sigma=2$ $\omega_d=5$ $\omega_n=5.39$

$t_r = 1.8/\cancel{\omega_d} = \boxed{\overset{0.33}{\cancel{0.36}}\text{ seg (SUBIDA)}}$ $\boxed{t_p=\pi/\omega_d = 0.63 \text{ seg (PICO)}}$

$t_s = 4.6/\sigma = \boxed{2.3\text{ seg (ESTABELECIMENTO)}}$

OVERSHOOT: $\xi = \cancel{\sigma}\,\sigma/\omega_n = 2/5.39 = 0.37 \longrightarrow$ APROXIMADAMENTE: $\boxed{M_p \simeq 25\%}$

(CÁLCULO MAIS PRECISO DO OVERSHOOT (OPCIONAL): $M_p = e^{\frac{-\pi\sigma}{\omega_d}} = e^{-0.4\pi} = 28.5\%$)

b) $$H(sI-F+GK)^{-1}G = \frac{1}{2}[21\ \ -19]\begin{bmatrix} s+16+3.5 & -13+0.5 \\ 17+3.5 & s-12+0.5\end{bmatrix}^{-1}\begin{bmatrix}1\\1\end{bmatrix}$$

$$H(sI-F+GK)^{-1}G = \frac{1}{2}[21\ \ -19]\frac{\begin{bmatrix} s-11.5 & 12.5 \\ -20.5 & s+19.5\end{bmatrix}}{s^2+8s+(19.5\times(-11.5))+20.5\times12.5}$$

$$H(sI-F+GK)^{-1}G = \frac{s+20}{s^2+8s+32}$$ PÓLOS: $s=-4\pm4j$ ZERO: $s=-20$

$\sigma=4$ $\omega_d=4$ $\omega_n=4\sqrt{2}=5.66$

$t_r = 1.8/5.66 = \boxed{0.32\text{ seg (SUBIDA)}}$ $\boxed{t_p = \pi/4 = 0.79\text{ seg (PICO)}}$

$t_s = 4.6/4 = \boxed{1.15\text{ seg (ESTABELECIMENTO)}}$

OVERSHOOT: $\xi = 4/(4\sqrt{2}) = 0.707 \longrightarrow$ APROXIMADAMENTE: $\boxed{M_p \simeq 5\%}$.

(CÁLCULO MAIS PRECISO (OPCIONAL): $M_p = e^{\frac{-\pi\sigma}{\omega_d}} = e^{-\pi} = 4.3\%$).

c) $\bar{N} = 1/\left(\lim_{s\to0} H(sI-F+GK)^{-1}G\right) \Rightarrow \bar{N} = 32/20 = \boxed{1.6}$

d) $$\mathcal{C} = \begin{bmatrix} 0.5 & -1.5 \\ 0.5 & -2.5\end{bmatrix} \quad \mathcal{C}^{-1} = -2\times\begin{bmatrix} -2.5 & 1.5 \\ -0.5 & 0.5\end{bmatrix} = \begin{bmatrix} \overset{(5)}{\cancel{5}} & -3 \\ 1 & -1\end{bmatrix}$$

$p_2 = [0\ \ 1]\mathcal{C}^{-1} = [1\ \ -1]$; $p_1 = p_2F = [1\ \ 1]$

$P = \begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$; $\boxed{T = \begin{bmatrix}0.5 & 0.5\\0.5 & -0.5\end{bmatrix}}$

QUESTÃO #3:

a) $G(s) = [0\ \ 1]\begin{bmatrix} s & 0\\ -1 & s\end{bmatrix}^{-1}\begin{bmatrix}1\\0\end{bmatrix} = \boxed{1/s^2}$

b) $D(s) = -K(sI-F+GK+LH)^{-1}L = -[3\ \ 2]\begin{bmatrix} s+3 & 27\\ -1 & s+10\end{bmatrix}^{-1}\begin{bmatrix}25\\10\end{bmatrix}$

$$D(s) = -\frac{[3 \;\; 2]\begin{bmatrix} s+10 & -27 \\ 1 & s+3 \end{bmatrix}}{s^2+13s+57}\begin{bmatrix} 25 \\ 10 \end{bmatrix} = -\frac{[3s+32 \quad 2s-75]}{s^2+13s+57}\begin{bmatrix} 25 \\ 10 \end{bmatrix}$$

$$\boxed{D(s) = -\frac{(95s+50)}{s^2+13s+57}}$$

c) SE $b=0 \Longrightarrow M=0 \Longrightarrow \dfrac{G(s)}{1-G(s)D(s)} = H(s)$

$$H(s) = \frac{\frac{1}{s^2}}{1+\frac{1}{s^2}\cdot\frac{95s+50}{s^2+13s+57}} = \frac{s^2+13s+57}{s^4+13s^3+57s^2+95s+50}$$

$$\alpha_c(s) = |sI-F+GK| = \begin{vmatrix} s+3 & 2 \\ -1 & s \end{vmatrix} = s^2+3s+2$$

$$\alpha_e(s) = |sI-F+LH| = \begin{vmatrix} \cancel{s+10} \; s & 25 \\ -1 & 10 \end{vmatrix} = s^2+10s+25$$

$$\begin{array}{ccccc|c} 1 & 10 & 25 & & & \\ 1 & & & & & 1 \\ 3 & 1 & & & & 13 \\ 2 & 3 & 1 & & & 57 \\ & 2 & 3 & & & 95 \\ & & 2 & & & 50 \end{array}$$

$\gamma(s)$: PÓLOS DE $D(s)$

$b(s)$: NUMERADOR DE $G(s)$

ENTÃO: $$\boxed{H(s) = \frac{(s^2+13s+57)\times(1)}{\underbrace{(s^2+3s+2)}_{\alpha_c(s)}\underbrace{(s^2+10s+25)}_{\alpha_e(s)}}}$$

PÓLOS DO CONTROLADOR — PÓLOS DO ERRO DE ESTIMAÇÃO

d) SE $\gamma(s) = |sI-F+\cancel{GK}+LH-\frac{MK}{\bar{N}}|$ E $M = G\bar{N}$ $(b=1)$, ENTÃO $\gamma(s) = \alpha_e(s)$.

PORTANTO: $$\boxed{H(s) = \frac{1}{s^2+3s+2}}$$

OBS.: PODE-SE ASSUMIR $\bar{N}=1$ NESTA QUESTÃO.

QUESTÃO #4:

a) $$G(s) = [1 \;\; 1]\begin{bmatrix} s & 0 \\ -1 & s \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = [1 \;\; 1]\begin{bmatrix} s & 0 \\ 1 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\cdot\frac{1}{s^2} = \frac{s+1}{s^2}$$

$\boxed{K = [4 \;\; 4]}$ (REPRESENTAÇÃO DADA NA FORMA CANÔNICA CONTROLÁVEL)

b) $\cancel{}$ $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\quad T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$T^{-1}FT = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \left[\begin{array}{c|c} 1 & -1 \\ 1 & -1 \end{array}\right] = F_z$$

$$F_{zbb} - LF_{zab} = -1+L \Longrightarrow |sI - F_{zbb} + LF_{zab}| = s+1-L = s+10$$

$$\boxed{L = -9}$$

c) $K_z = KT = [4 \;\; 4]\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = [4 \,|\, 0]$

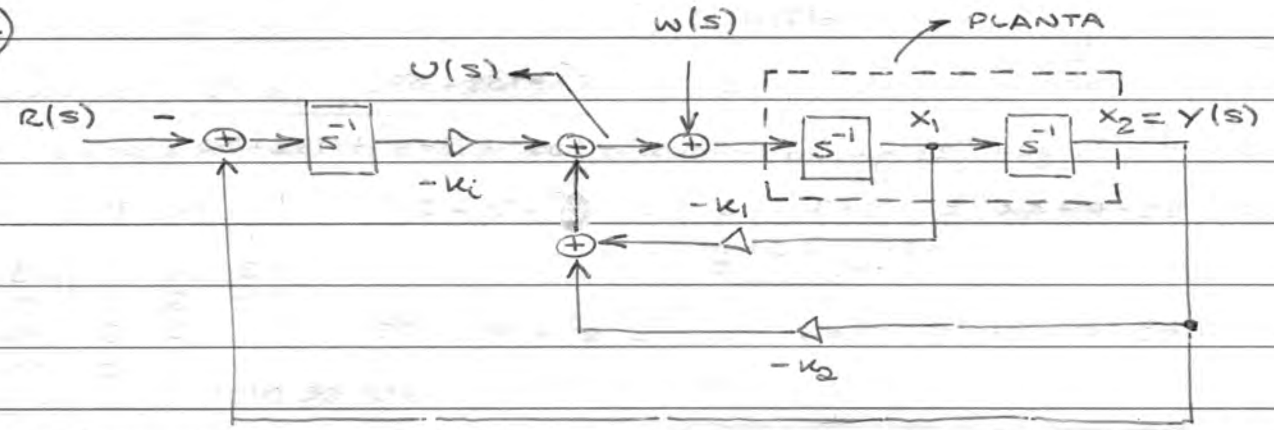
$H_r = -K_{zb} = 0$ (F_r E G_r NÃO PRECISAM SER CALCULADOS)

$J_r = -K_{za} - K_{zb}L = -4 \Longrightarrow \boxed{D(s) = -4}$

d) $\dfrac{G(s)}{1-G(s)D(s)} = \dfrac{\frac{s+1}{s^2}}{1+\frac{4(s+1)}{s^2}} = \boxed{\dfrac{s+1}{s^2+4s+4}}$ $\boxed{\bar{N}=4}$

QUESTÃO #5:

a)



b) $\begin{vmatrix} s & 0 & 1 \\ k_i & s+k_1 & k_2 \\ 0 & -1 & s \end{vmatrix} = s^3 + k_1 s^2 + k_2 s + k_i$

1	2	2	1
1	1		3
	1	1	4
		1	2

$\alpha_c(s) = (s+1-j)(s+1+j)(s+1) = (s^2+2s+2)(s+1) = s^3+3s^2+4s+2$

ENTÃO: $\boxed{k_i = 2 \;;\; k_1 = 3 \;;\; k_2 = 4}$

c) $\dfrac{Y(s)}{R(s)} = [0\ 0\ 1] \begin{bmatrix} s & 0 & 1 \\ 2 & s+3 & 4 \\ 0 & -1 & s \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \boxed{\dfrac{2}{s^3+3s^2+4s+2}}$ (*)

d) $\dfrac{Y(s)}{W(s)} = [0\ 0\ 1] \begin{bmatrix} s & 0 & 1 \\ 2 & s+3 & 4 \\ 0 & -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \boxed{\dfrac{s}{s^3+3s^2+4s+2}}$ (*)

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(*) $2 = (-1)^{1+3} \begin{vmatrix} 2 & s+3 \\ 0 & -1 \end{vmatrix}$ (REMOVER LINHA 1 E COLUNA 3)

$s = (-1)^{2+3} \cdot \begin{vmatrix} s & 0 \\ 0 & -1 \end{vmatrix}$ (REMOVER LINHA 2 E COLUNA 3)

NOTE QUE: $\lim_{s \to 0} \dfrac{Y(s)}{R(s)} = 1$ E $\lim_{s \to 0} \dfrac{Y(s)}{W(s)} = 0$

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