Universidade Federal do Rio de Janeiro
Escola Politécnica

Data: 25 / 04 / 2008

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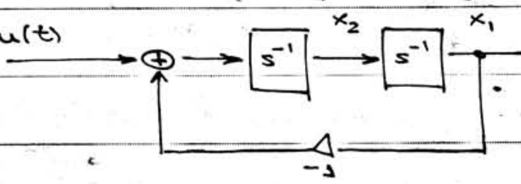
Aluno: GABARITO DA PROVA PARCIAL #1

Disciplina: CONTROLE LINEAR II·A — Turma: EEL760

Professor: GABRIEL

QUESTÃO #1:



a) $$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \;\; 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b) $$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \implies \frac{Y(s)}{R(s)} = \frac{[1 \;\; 0] \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2+5s+6} = \frac{1}{s^2+5s+6}$$

$$y = [1 \;\; 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

c) $$Y(s) = \frac{[1 \;\; 0] \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2+5s+6} = \frac{s+6}{s^2+5s+6} = \frac{4}{s+2} + \frac{(-3)}{s+3} \implies y(t) = \left(4e^{-2t} - 3e^{-3t}\right)u(t)$$

d) $$Y(s) = \frac{1}{s} \cdot \frac{1}{s^2+5s+6} = \frac{1/6}{s} + \frac{-1/2}{s+2} + \frac{1/3}{s+3} \implies y(t) = \left(\frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t}\right)u(t)$$

QUESTÃO #2:

a) $$G(s) \overset{(FCM)}{=} \frac{19}{s+1} - \frac{18}{s+2} = \frac{s+20}{s^2+3s+2} \qquad \left(F_{cm} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \quad G_{cm} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

b) $t_p = \pi/3 \implies \omega_d = 3$ rad/seg ; $M_p = 0.05 \implies \theta = \operatorname{asin}(0.7) \simeq 45^\circ \implies \sigma = -3$

PÓLOS DESEJADOS: $-3 \pm 3j \implies \alpha_c(s) = (s+3+3j)(s+3-3j) = s^2 + 6s + 18$

$$|sI - F_{cm} + G_{cm}K| = \begin{vmatrix} s+1+k_1 & k_2 \\ k_1 & s+2+k_2 \end{vmatrix} = s^2 + (k_1+k_2+3)s + 2k_1 + k_2 + 2$$

$$\begin{cases} k_1 + k_2 = 3 \\ 2k_1 + k_2 = 16 \end{cases} \implies k_1 = 13 \implies k_2 = -10 \qquad (K = [13 \;\; -10])$$

c) $$sI - F_{cm} + G_{cm}K = \begin{bmatrix} s+14 & -10 \\ 13 & s-8 \end{bmatrix}$$

(ok.)

$$\frac{Y(s)}{R(s)} = \frac{[19 \;\; -18] \begin{bmatrix} s-8 & 10 \\ -13 & s+14 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2+6s+18} = \frac{[19 \;\; -18] \begin{bmatrix} s+2 \\ s+1 \end{bmatrix}}{s^2+6s+18} = \frac{s+20}{s^2+6s+18} = \frac{b(s)}{\alpha_c(s)}$$

d) SE $\bar{N} = 1$, ENTÃO $\lim_{s \to 0} \frac{Y(s)}{R(s)} = \frac{20}{18}$. PORTANTO, $\boxed{\bar{N} = 0.9}$ PARA QUE $\lim_{s \to 0} \frac{Y(s)}{R(s)} = 1.0$

QUESTÃO #3:

$$(P = T^{-1})$$

a) $F = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$; $G = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $H = [0\ 0\ 1]$; $T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$F_z = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{array}{c} \begin{array}{cc} F_{zaa} & F_{zab} \end{array} \\ \left[\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cc} F_{zba} & F_{zbb} \end{array} \end{array}$$

$$G_z = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{array}{c} \left[\begin{array}{c} 0 \\ \hline 0 \\ 1 \end{array}\right] \end{array}\begin{array}{l} G_{za} \\ \\ G_{zb} \end{array} \quad ; \quad H_z = [0\ 0\ 1]\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = [1\ 0\ 0]$$

b) $\left| sI - F_{zbb} + LF_{zab} \right| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix}[1\ 0] \right| = \begin{vmatrix} s+\ell_1 & -1 \\ \ell_2 & s \end{vmatrix} = s^2 + \ell_1 s + \ell_2$

$\alpha_e(s) = s^2 + 20s + 100 \implies \ell_1 = 20$ E $\ell_2 = 100 \implies L = \begin{bmatrix} 20 \\ 100 \end{bmatrix}$

c) $K_z = K_{cc}T = [3\ 3\ 1]\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = [\underbrace{1}_{K_{za}} | \underbrace{3\ 3}_{K_{zb}}]$

$$F_r = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 20 \\ 100 \end{bmatrix}[1\ 0] - \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)[3\ 3] = \begin{bmatrix} -20 & 1 \\ -103 & -3 \end{bmatrix}$$

$$G_r = \begin{bmatrix} -20 & 1 \\ -103 & -3 \end{bmatrix}\begin{bmatrix} 20 \\ 100 \end{bmatrix} - \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)[1] = \begin{bmatrix} -300 \\ -2361 \end{bmatrix}$$

$H_r = [-3\ -3]$ E $J_r = -1 - [3\ 3]\begin{bmatrix} 20 \\ 100 \end{bmatrix} = -361$

$$D(s) = H_r(sI - F_r)^{-1}G_r + J_r = [-3\ -3]\frac{\begin{bmatrix} s+3 & 1 \\ -103 & s+20 \end{bmatrix}}{s^2 + 23s + 163}\begin{bmatrix} -300 \\ -2361 \end{bmatrix} + (-361)$$

$$D(s) = \frac{(-3) \times [s-100 \quad s+21]\begin{bmatrix} -300 \\ -2361 \end{bmatrix} + (-361)(s^2 + 23s + 163)}{s^2 + 23s + 163} = \boxed{\frac{-361s^2 - 320s - 100}{s^2 + 23s + 163}}$$

d) $$\frac{Y(s)}{R(s)} = \frac{\frac{1}{s^3}}{1 + \frac{361s^2 + 320s + 100}{s^5 + 23s^4 + 163s^3}} = \frac{s^2 + 23s + 163}{s^5 + 23s^4 + 163s^3 + 361s^2 + 320s + 100} \overset{(*)}{=} \frac{\gamma(s)b(s)}{\alpha_c(s)\alpha_e(s)}$$

(*) PARA VERIFICAR ISTO, NOTE QUE: $b(s) = 1$; $\alpha_e(s) = s^2 + 20s + 100$

E QUE $K_{cc} = [3\ 3\ 1] \implies \alpha_c(s) = s^3 + 3s^2 + 3s + 1$

1	20	100	$(\alpha_c\alpha_e)$
1			1
3	1		23
3	3	1	163
1	3	3	361
	1	3	320
		1	100

QUESTÃO #4:

a) $e^{x_{20}} - 1 = 0 \implies x_{20} = 0$

$x_{10} = 0$

→ PONTO DE EQUILÍBRIO: $\begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

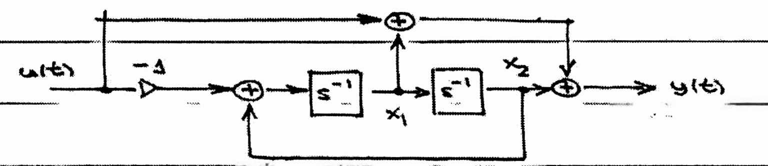
b) $\frac{\partial f_1}{\partial x_1} = 0$ $\quad \left.\frac{\partial f_1}{\partial x_2}\right|_{x_2 = x_{20} = 0} = \left. e^{x_{20}} \right|_{x_{20}=0} = 1$ $\quad \frac{\partial f_1}{\partial u} = -1$

$\frac{\partial f_2}{\partial x_1} = 1$ $\quad \frac{\partial f_2}{\partial x_2} = 0$ $\quad \frac{\partial f_2}{\partial u} = 0$

$\implies F = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; $\implies G = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(OBS.: $y = x_1 + x_2 + u \implies H = [1\ 1]$ E $J = 1$)

c) $\begin{cases} x_1' = x_2 - u \\ x_2' = x_1 \\ y = x_1 + x_2 + u \end{cases}$



$$G(s) = [1\ 1]\,\frac{\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}}{s^2-1}\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 1 = \frac{-\cancel{(s+1)}}{\cancel{(s+1)}(s-1)} + 1 = \boxed{\frac{s-2}{s-1}}$$

G(s)

d) $\mathcal{C} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$; $\det \mathcal{C} = 1$ => SISTEMA CONTROLÁVEL

$\mathcal{O} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$; $\det \mathcal{O} = 0$ => SISTEMA NÃO-OBSERVÁVEL.

QUESTÃO #5:

a) $\boxed{G(s) = \frac{s+1}{s^2+2s+2}}$ (OBTIDA DIRETAMENTE DA FCO) (FCO) $F_{co} = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}$

b) $B = -2Y + 6(4Y + X_c)$; $-4(X_c + 4Y) + 6Y - 3U = sX_c$

$B = -2Y + 24Y + \frac{(-60Y + 18U)}{s+4}$ $\qquad X_c(s+4) = -10Y - 3U$

$B = \frac{22(s+4)Y - (60Y + 18U)}{s+4}$ $\qquad X_c = \frac{-10Y - 3U}{s+4}$

$\boxed{B = \frac{(22s+28)Y - 18U}{s+4}}$ ⟶ B EM FUNÇÃO DE Y(s) E U(s).

c) $U = -\left(\frac{(22s+28)Y - 18U}{s+4} + \frac{16}{s}(Y - R)\right)$

$U(s+4)s = -\left((22s^2 + 28s)Y - 18sU + (16s+64)Y - (16s+64)R\right)$

$\boxed{U(s-14)s = -\left((22s^2 + 44s + 64)Y - (16s+64)R\right)}$ ⟶ U EM FUNÇÃO DE Y(s) E R(s).

⟶ $G(s)\left(U(s) + W(s)\right) = Y(s)$

$\frac{(16s+64)R - (22s^2+44s+64)Y}{s^2-14s} + W = \frac{s^2+2s+2}{s+1}Y$ $\qquad (s^2-14s)Y$

$(16s+64)(s+1)R - (22s^2+44s+64)(s+1)Y + (s^2-14s)(s+1)W = (s^2+2s+2)$

$\boxed{(s^4 + 10s^3 + 40s^2 + 80s + 64)Y = \overset{16}{\cancel{1}}(s+\overset{4}{\cancel{14}})(s+1)R + s(s-14)(s+1)W}$

22	44	64		
1	1			
	1	1		
		1	1	
			1	1

	1	2	2	
22	-14	1		
66		-14	1	
108			-14	1
64				-14 1

	1	-12	-26	-28	0
1		22	66	108	64
-12	1	10	40	80	64
-26					
-28					

$(16s^2 + 80s + 64)$

d) $\boxed{\frac{Y(s)}{R(s)} = \frac{\overset{16}{\cancel{1}}s^2 + \overset{80}{\cancel{88}}s + 64}{s^4 + 10s^3 + 40s^2 + 80s + 64}}$ $\qquad \boxed{\frac{Y(s)}{W(s)} = \frac{s^3 - 13s^2 - 14s}{s^4 + 10s^3 + 40s^2 + 80s + 64}}$

$sI - F_{bb} + LF_{ab} = s + L \implies \alpha_e(s) = s + 4$

1	10	40	80	64	1 4
0	6	40	80	64	1 6 16 16
	0	16	80	64	
		0	16	64	
			0	0	

ENTÃO : $\boxed{\alpha_c(s) = s^3 + 6s^2 + 16s + 16}$

(RAÍZES ⟶ -2 ; $-2-2j$; $-2+2j$)