Universidade Federal do Rio de Janeiro

Escola Politécnica

DATA: 28/9/2007

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Aluno: GABARITO DA PROVA PARCIAL #1

Disciplina: EEL760 — CONTROLE II

Turma: 2007/2

Professor: GABRIEL

QUESTÃO #1:

a) $G(s) = \dfrac{[1 \;\; 1]\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}}{s^2+3s+2}\begin{bmatrix} 4 \\ -4 \end{bmatrix} = \dfrac{4}{(s+1)(s+2)}$

$\underset{\circ}{Y}(s) = \dfrac{4}{s^2(s+1)(s+2)} = \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{s+1} + \dfrac{D}{s+2} = \dfrac{2}{s^2} + \dfrac{(-3)}{s} + \dfrac{4}{s+1} - \dfrac{1}{s+2}$

$A = 4/2 = 2 \qquad B = \left.\dfrac{-4(2s+3)}{((s+1)(s+2))^2}\right|_{s=0} = -3 \qquad C = \left.\dfrac{4}{s^2(s+2)}\right|_{s=-1} = 4 \qquad D = \left.\dfrac{4}{s^2(s+1)}\right|_{s=-2} = -1$

$2(\cancel{s^2}+\cancel{3s}+\cancel{2}) - 3s(\cancel{s^2}+\cancel{3s}+\cancel{2}) + 4s^2(\cancel{s}+\cancel{2}) - s^2(\cancel{s}+\cancel{1})$

$y_0(t) = (2t - 3 + 4e^{-t} - e^{-2t})\,u(t)$

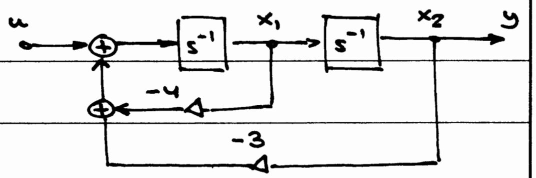
b) $Y_1(s) = \dfrac{[1 \;\; 1]\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}}{s^2+3s+2}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \dfrac{\cancel{s+2}}{(s+1)\cancel{(s+2)}} = \dfrac{1}{s+1} \quad \therefore \quad y_1(t) = e^{-t}u(t)$

c) $Y_2(s) = \dfrac{[1 \;\; 1]\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}}{s^2+3s+2}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \dfrac{\cancel{s+1}}{\cancel{(s+1)}(s+2)} = \dfrac{1}{s+2} \quad \therefore \quad y_2(t) = e^{-2t}u(t)$

d) $y_3(t) = y_0(t) - y_1(t) + y_2(t) = (2t - 3 + 3e^{-t})\,u(t)$

QUESTÃO #2:

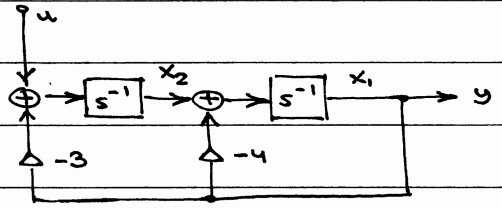
a) $G(s) = \dfrac{1}{s^2+4s+3}$ FCC: $x' = \begin{bmatrix} -4 & -3 \\ 1 & 0 \end{bmatrix}x + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u$



$y = [0 \;\; 1]\,x$

$x_2(0) = y(0) = 1 \quad$ e $\quad x_1(0) = x_2'(0) = y'(0) = 2 \implies x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

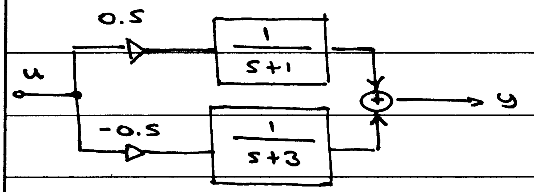
b) FCO: $x' = \begin{bmatrix} -4 & 1 \\ -3 & 0 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$



$y = [1 \;\; 0]\,x$

$x_1(0) = y(0) = 1 \quad$ e $\quad \underbrace{x_1'(0)}_{2} = -4\underbrace{x_1(0)}_{1} + x_2(0) \implies x_2(0) = 6 \implies x(0) = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$

c) FCM: $G(s) = \frac{1}{(s+1)(s+3)} = \frac{1/2}{s+1} + \frac{(-1/2)}{s+3}$ $\qquad x' = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} u$



$y = [1 \;\; 1]\, x$

$y(0) = 1 \longrightarrow \boxed{x_1(0) + x_2(0) = 1}$

$y'(0) = 2 \longrightarrow x_1'(0) + x_2'(0) = 2 \longrightarrow \boxed{-x_1(0) - 3x_2(0) = 2}$

$\underbrace{\begin{bmatrix} -5 & -3 \\ 1 & 0 \end{bmatrix}}$ $\qquad x_2(0) = -3/2$ E $x_1(0) = 5/2 \implies x(0) = \begin{bmatrix} 2.5 \\ -1.5 \end{bmatrix}$

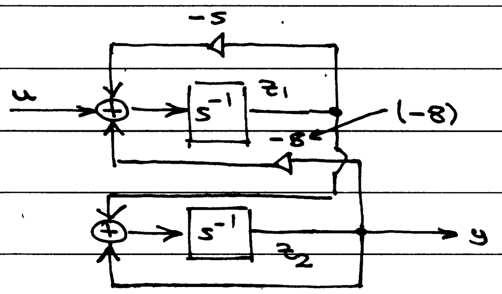
d) $F = T^{-1} F_{cc} T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & -3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -8 \\ 1 & 1 \end{bmatrix}$

$G = T^{-1} G_{cc} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\qquad H = H_{cc} T = [0 \;\; 1] \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = [0 \;\; 1]$

$z(0) = T^{-1} x(0) \implies z(0) = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \implies z(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$z' = \begin{bmatrix} -5 & -8 \\ 1 & 1 \end{bmatrix} z + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

$y = [0 \;\; 1]\, z$



QUESTÃO #3:

a) $F - GK = \begin{bmatrix} -2-k_1 & -1-k_2 \\ 1 & 0 \end{bmatrix}$; $|sI - F + GK| = \begin{vmatrix} s+k_1+2 & k_2+1 \\ -1 & s \end{vmatrix} = s^2 + (k_1+2)s + k_2 + 1$

ENTÃO $k_1 = 2$ E $k_2 = 3 \implies K = [2 \;\; 3]$

b) $|sI - F_{bb} + LF_{ab}| = s - 0 - L = s + 10 \implies L = -10$

c) $F_r = 0 - 10 - 10 \times 3 = -40$ $\qquad D(s) = \frac{-3 \times 361}{s+40} + 28 = \frac{28s + 37}{s+40}$

$G_r = 400 + 1 - 20 - 10 \times 2 = 361$

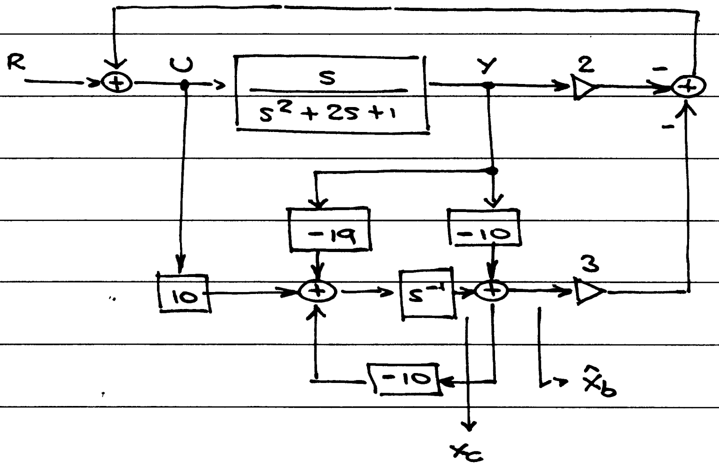
$H_r = -3$

$J_r = -2 + 30 = 28$

d) $F_{ba} - LF_{aa} = -19$

$G_b - L G_a = 10$

$F_{bb} - LF_{ab} = -10$



FUNÇÃO DE TRANSFERÊNCIA $\frac{Y(s)}{R(s)}$:

(1) $sX_c = 10(R - 2Y - 3\hat{X}_b) - 19Y - 10\hat{X}_b$

$\hat{X}_b = X_c - 10Y \Rightarrow X_c = \hat{X}_b + 10Y$

$s(\hat{X}_b + 10Y) = 10(R - 2Y - 3\hat{X}_b) - 19Y - 10\hat{X}_b$

$\hat{X}_b(s+40) = Y(-10s - 39) + 10R$ //

(2) $(R - 2Y - 3\hat{X}_b) \cdot \frac{s}{s^2+2s+1} = Y$

$Y(s^2+4s+1) = Rs - 3s\hat{X}_b$

$Y(s^2+4s+1)(s+40) = Rs(s+40) - 3s\boxed{(s+40)\hat{X}_b}$

$Y(s^2+4s+1)(s+40) = Rs(s+40) - 3sY(-10s-39) - 3s.10R$

$Y(s^3+44s^2+161s+40-30s^2-117s) = R(s^2+10s)$

$Y\underbrace{(s^3+14s^2+44s+40)}_{(s+10)(s^2+4s+4)} = Rs(s+10) \Rightarrow \frac{Y(s)}{R(s)} = \frac{s}{s^2+4s+4}$ //

QUESTÃO #4:

a) $$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -k_2 & -1-k_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \cdot r + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \omega$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

b) $$\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ k_1 & k_2 & s+k_3+1 \end{vmatrix} = s^3 + (k_3+1)s^2 + k_2 s + k_1 = s^3 + 6s^2 + 12s + 8$$

$k_1 = 8$; $k_2 = 12$; $k_3 = 5$

c) $$x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -12 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cancel{\omega}^{\,0}$$

$y = [0\ 0\ 1]\, x$

$$\frac{Y(s)}{R(s)} = [0\ 0\ 1] \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 8 & 12 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \frac{12s+8}{s^3+6s^2+12s+8}$$

d) $$x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -12 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \cancel{r}^{\,0} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega$$

$y = [0\ 0\ 1]\, x$

$$\frac{Y(s)}{W(s)} = [0\ 0\ 1] \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 8 & 12 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{s^2}{s^3+6s^2+12s+8}$$

QUESTÃO #5:

a) $\ln(x_{10}+x_{20}) = 0 \Rightarrow x_{10}+x_{20} = 1.$

$e^{x_{10}-x_{20}} = 1 \Rightarrow x_{10}-x_{20} = 0$

$\Rightarrow \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

b) $f_1(x_1,x_2,u) = \ln(x_1+x_2)$

$\left.\dfrac{\partial f_1}{\partial x_1}\right|_{x_{10},x_{20}} = \dfrac{1}{x_{10}+x_{20}} = 1 \; ; \; \left.\dfrac{\partial f_1}{\partial x_2}\right|_{x_{10},x_{20}} = \dfrac{1}{x_{10}+x_{20}} = 1 \; ; \; \dfrac{\partial f_1}{\partial u} = 0$

$f_2(x_1,x_2,u) = e^{x_1-x_2}+u$

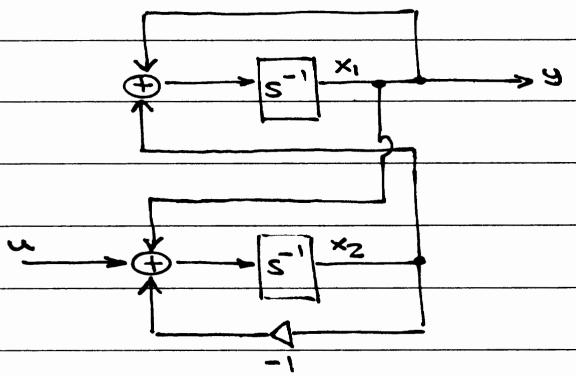
$\left.\dfrac{\partial f_2}{\partial x_1}\right|_{x_{10},x_{20}} = e^{x_{10}-x_{20}} = 1 \; ; \; \left.\dfrac{\partial f_2}{\partial x_2}\right|_{x_{10},x_{20}} = -e^{x_{10}-x_{20}} = -1 \; ; \; \dfrac{\partial f_2}{\partial u} = 1$

$F = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad G = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad y = x_1 \Rightarrow H = [1 \; 0]$

$x' = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = [1 \; 0]\, x$

c)



d) $|sI - F| = \begin{vmatrix} s-1 & -1 \\ -1 & s+1 \end{vmatrix} = s^2 - 2 = (s+\sqrt{2})(s-\sqrt{2}) \Rightarrow$ SISTEMA INSTÁVEL

$s = -\sqrt{2}$ ou $s = \sqrt{2}$

$\mathcal{C} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \; ; \; |\mathcal{C}| = -1 \Rightarrow$ SISTEMA CONTROLÁVEL

$\Theta = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \; ; \; |\Theta| = 1 \Rightarrow$ SISTEMA OBSERVÁVEL