Universidade Federal do Rio de Janeiro

Escola Politécnica

DATA: 27/04/2007

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Aluno: GABARITO —— PROVA PARCIAL # 1

Disciplina: CONTROLE LINEAR II-A

Turma: EEL760

Professor: JOSÉ GABRIEL R.C. GOMES

QUESTÃO # 1:

a) $Y_{ZI}(s) = [0\ \ 1]\begin{bmatrix} s-1 & 0 \\ -1 & s+1 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = [0\ \ 1]\underbrace{\begin{bmatrix} s+1 & 0 \\ 1 & s-1 \end{bmatrix}}_{s^2-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \dfrac{s}{s^2-1}$

$Y_{ZI}(s) = \dfrac{A}{s+1} + \dfrac{B}{s-1} = \dfrac{1/2}{s+1} + \dfrac{1/2}{s-1}$ $\qquad y_{ZI}(t) = \left(\dfrac{e^{-t}+e^{t}}{2}\right)u(t)$

b) $Y_{ZS}(\cancel{x}\,s) = [0\ \ 1]\begin{bmatrix} s-1 & 0 \\ -1 & s+1 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\cdot\dfrac{1}{s} = [0\ \ 1]\underbrace{\begin{bmatrix} s+1 & 0 \\ 1 & s-1 \end{bmatrix}}_{s(s^2-1)}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \dfrac{1}{s(s+1)(s-1)}$

$Y_{ZS}(\cancel{x}\,s) = \dfrac{A}{s+1} + \dfrac{B}{s-1} + \dfrac{C}{s} = \dfrac{1/2}{s+1} + \dfrac{1/2}{s-1} - \dfrac{1}{s}$ $\qquad \left(A = \dfrac{1}{s(s-1)}\Big|_{s=-1} = \dfrac{1}{2}, \dots\right)$

$y(t) = y_{ZI}(t) + y_{ZS}(t) = \left(\dfrac{e^{-t}+e^{t}}{2} + \dfrac{e^{-t}+e^{t}}{2} - 1\right)u(t)$; $y(t) = \cancel{(1-e^{-t}-e^{t})u(t)}$ $(e^{t}+e^{-t}-1)\,u(t)$

c) $\mathcal{C} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$; $\mathcal{C}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$; $p_2 = [0\ \ 1]\,\mathcal{C}^{-1} = [0\ \ 1]$; $p_1 = [0\ \ 1]\,F = [1\ \ -1]$

ENTÃO $P = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ e $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$F_{cc} = T^{-1}FT = \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}}_{\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$G_{cc} = T^{-1}G = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$H_{cc} = HT = [0\ \ 1]\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = [0\ \ 1]$

d) $\mathcal{O} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$; $\mathcal{O}^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$; $t_2 = \mathcal{O}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$; $t_1 = F\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

ENTÃO $T = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ e $P = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$F_{co} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}}_{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$G_{co} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$H_{co} = [0\ \ 1]\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = [1\ \ 0]$

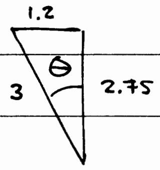
QUESTÃO #2:

a) FCO: $x' = \begin{bmatrix} -2.4 & 1 \\ -9 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ ← r $(k_1 = k_2 = 0)$

$y = [1 \;\; 0]\, x$

$$\frac{Y(s)}{R(s)} = \frac{1}{s^2 + 2.4s + 9}$$

b) PÓLOS: $-1.2 \pm 2.75j$

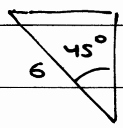


$\sin\theta = \frac{1.2}{3} = 0.4 \longrightarrow$ OVERSHOOT: 25%

$t_r = \frac{1.8}{3} = 0.6$ seg

c) $t_r = 0.3$ seg $\longrightarrow \omega_n = 6$

OVERSHOOT 5% $\longrightarrow \theta = 45^\circ$



PÓLOS: $-3\sqrt{2} \pm 3\sqrt{2}\, j$

$$\alpha_c(s) = (s + 3\sqrt{2} + 3\sqrt{2}j)(s + 3\sqrt{2} - 3\sqrt{2}j) = s^2 + \overbrace{6\sqrt{2}}^{8.49} s + 36$$

$$|sI - F + GK| = \left| \begin{bmatrix} s+2.4 & -1 \\ 9 & s \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1 \;\; k_2] \right| = \begin{vmatrix} s+2.4 & -1 \\ 9+k_1 & s+k_2 \end{vmatrix}$$

$$= s^2 + \underbrace{(k_2 + 2.4)}_{8.49} s + \underbrace{k_1 + 2.4k_2 + 9}_{36}$$

$\hookrightarrow k_2 = 6.09 \quad \longrightarrow \quad k_1 = 12.38$

$K = [12.38 \;\; 6.09]$

d) $x' = \begin{bmatrix} -2.4 & 1 \\ -21.38 & -6.09 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$

$y = [1 \;\; 0]\, x$

$$\frac{Y(s)}{R(s)} = [1 \;\; 0] \begin{bmatrix} s+2.4 & -1 \\ 21.38 & s+6.09 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [1 \;\; 0] \frac{\begin{bmatrix} \overset{s+6.09}{\cancel{s+2.4}} & 1 \\ -21.38 & s+2.4 \end{bmatrix}}{s^2 + 8.49s + 36} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{Y(s)}{R(s)} = \frac{1}{s^2 + 8.49s + 36} \qquad \left(\frac{b(s)}{\alpha_c(s)} \text{ ou.} \right)$$

QUESTÃO #3:

a) $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; $T^{-1}FT = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}}_{\begin{bmatrix} 1 & 0 \\ -2 & -2 \end{bmatrix}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \left[\begin{array}{c|c} \overset{F_{aa}}{0} & \overset{F_{ab}}{1} \\ \underset{F_{ba}}{-2} & \underset{F_{bb}}{-2} \end{array}\right] = F_z$

$G_z = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$H_z = [0 \;\; 1] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = [1 \;\; 0]$

b) $|sI - F_{bb} + LF_{ab}| = s + 2 + L = s + 10 \implies L = 8$

c) $K_z = KT \implies K_z = [2 \;\; 1] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = [1 \;\; 2]$

$F_r = -2-8-(1)\times 2 = -12$

$G_r = -12\times 8 - 2 - (1)\times 1 = -99$

$H_r = -2$

$J_r = -1-16 = -17$

d) $D(s) = H_r(sI - F_r)^{-1}G_r + J_r = \dfrac{-2\times(-99)}{s+12} - 17 = \dfrac{-(17s+6)}{s+12}$

VERIFICAÇÃO: $\alpha_c(s).\alpha_e(s) = (s^2+4s+3)(s+10) = s^3+14s^2+43s+30$

s^2+2s+2 (PLANTA)

PORQUE: $\downarrow +2 \quad \downarrow +1$ (VETOR K_x)

s^2+4s+3

$$\frac{G(s)}{1-G(s)D(s)} = \frac{\dfrac{1}{s^2+2s+2}}{1+\dfrac{1}{s^2+2s+2}\cdot\dfrac{17s+6}{s+12}} = \frac{s+12}{s^3+2s^2+2s+12s^2+24s+24+17s+6}$$

$= \alpha_c(s)\alpha_e(s)$, CONFORME ESPERADO.

QUESTÃO #4:

a)
$$\begin{cases} x_1' = -k_1\hat{x}_1 - k_2\hat{x}_2 + r \\ x_2' = x_1 \\ \hat{x}_1' = \ell_1 x_2 - \ell_1\hat{x}_2 + m_1 r - k_1\hat{x}_1 - k_2\hat{x}_2 \\ \hat{x}_2' = \ell_2 x_2 - \ell_2\hat{x}_2 + m_2 r + \hat{x}_1 \\ y = x_2 \end{cases}$$

<u>OBS.:</u> PLANTA: $x_1' = u$, $x_2' = x_1$, $y = x_2$ $\Rightarrow$ $x' = \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}}_{F} x + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{G} u$

$y = \underbrace{[0 \quad 1]}_{H} x$

b) $|sI - F + GK| = \begin{vmatrix} s+k_1 & k_2 \\ -1 & s \end{vmatrix} = s^2 + k_1 s + k_2 = s^2+2s+1$

$k_1 = 2$ E $k_2 = 1$ $\Rightarrow$ $K = [2 \quad 1]$

c) $|sI - F + LH| = \begin{vmatrix} s & \ell_1 \\ -1 & s+\ell_2 \end{vmatrix} = s^2 + \ell_2 s + \ell_1 = s^2 + 10s + 25$

$\ell_2 = 10$ E $\ell_1 = 25$ $\Rightarrow$ $L = \begin{bmatrix} 25 \\ 10 \end{bmatrix}$

d) $|sI - F + GK + LH - MK| = \left| \begin{bmatrix} s+2 & 26 \\ -1 & s+10 \end{bmatrix} - \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} [2 \quad 1] \right|$ $(\bar{N}=1)$

$F - GK - LH = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 25 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} -2 & -26 \\ 1 & -10 \end{bmatrix}$

$|sI - F + GK + LH - MK| = \begin{vmatrix} s+2-2m_1 & 26-m_1 \\ -1-2m_2 & s+10-m_2 \end{vmatrix} = s^2 + (12-2m_1-m_2)s + 46 - 21m_1 + 50m_2$

$= s^2 + 6s + 5$

ENTÃO : $\begin{cases} 2m_1 + m_2 = 6 \\ -21m_1 + 50m_2 = -41 \end{cases} \longrightarrow -121m_1 = -341 \longrightarrow m_1 = 2.82$

$m_2 = 0.36$

$$M = \begin{bmatrix} 2.82 \\ 0.36 \end{bmatrix}$$

QUESTÃO #5 :

a) $x_1' = f(x_1, u) = x_1^2 + u$

$f(x_{10}, u_0) = 0 \Longrightarrow x_{10}^2 - 1 = 0 \xrightarrow{x_{10} < 0} x_{10} = -1$

b) $\left.\dfrac{\partial f}{\partial x_1}\right|_{x_1 = x_{10}} = \left. 2x_1 \right|_{x_1 = x_{10}} = -2 \; ; \; F = -2$

$\dfrac{\partial f}{\partial u} = 1 \longrightarrow G = 1$

$y = 2x \longrightarrow H = 2$

c) $F_i = \begin{bmatrix} 0 & 2 \\ 0 & -2 \end{bmatrix} \qquad G_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

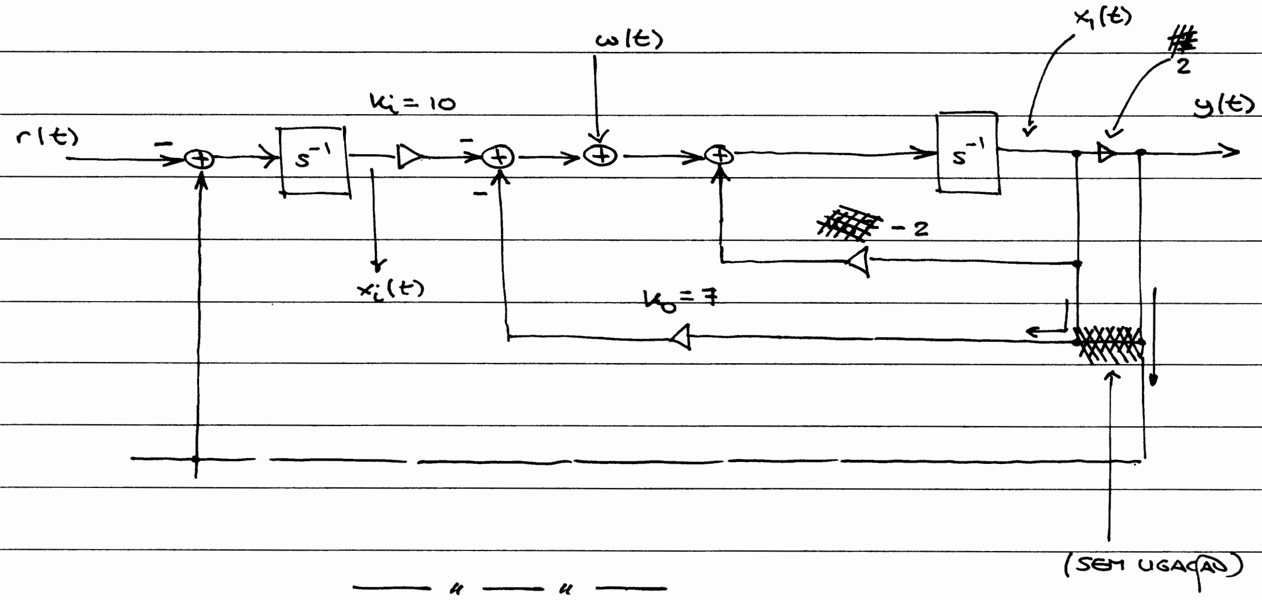
$|sI - F_i + G_i K| = \begin{vmatrix} s & -2 \\ K_1 & s + K_2 + 2 \end{vmatrix} = s^2 + (K_2 + 2)s + 2K_1$

$\alpha_c(s) = (s+4)(s+5) = s^2 + 9s + 20 \Longrightarrow K_1 = 10 = K_i$

$K_2 = 7 = K_0$

$K = [K_i \;\; K_0] = [10 \;\; 7]$

d)



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