GABARITO — P1 DE 2006/2 — 060929

① a) $Y(s) = \frac{1}{s}\left(\begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} s & -1 \\ -1 & s \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1 \right) = \frac{1}{s}\left(\frac{\cancel{s-1}}{\underbrace{\cancel{s^2-1}}_{s+1}} + 1 \right) = \frac{s+2}{s(s+1)}$

$Y(s) = \frac{2}{s} + \frac{(-1)}{s+1} \longrightarrow y(t) = (2 - e^{-t})u(t)$

b) $Y(s) = \begin{pmatrix} 1 & -1 \end{pmatrix} \frac{\begin{pmatrix} s & 1 \\ 1 & s \end{pmatrix}}{s^2-1} \begin{pmatrix} a \\ b \end{pmatrix} = \frac{\cancel{(s-1)}(a-b)}{\underbrace{\cancel{s^2-1}}_{s+1}} = \frac{a-b}{s+1}$

$y(t) = (a-b)e^{-t}u(t)$

c) $X_2(s) = \begin{pmatrix} 0 & 1 \end{pmatrix} \frac{\begin{pmatrix} s & 1 \\ 1 & s \end{pmatrix}}{s^2-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{s^2-1} = \frac{-1/2}{s+1} + \frac{1/2}{s-1}$

$x_2(t) = \left(-\frac{1}{2}e^{-t} + \frac{1}{2}e^{t} \right) u(t)$

d) $\mathcal{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \mathcal{O} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $\quad \begin{vmatrix} s & -1 \\ -1 & s \end{vmatrix} = s^2 - 1 = 0$

CONTROLÁVEL NÃO-OBSERVÁVEL $s = \pm 1 \longrightarrow$ INSTÁVEL

② a) $G(s) = \begin{pmatrix} 1 & 20 \end{pmatrix} \begin{pmatrix} s+2 & 2 \\ -1 & s \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{s+20}{s^2+2s+2}$

PÓLOS : $s = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm j \rightarrow \sigma = 1 \ ; \ \omega_d = 1 \ ; \ \omega_n = \sqrt{2} \ ; \ \xi = 0.7$

$\xi = 0.7 \longrightarrow M_p = 5\%$ $\quad t_s = \frac{4.6}{1} = 4.6$ seg

$t_r = \frac{1.8}{\sqrt{2}} = 1.04$ seg $\quad t_p = \frac{\pi}{1} = 3.14$ seg

b) $\alpha_1 = 2 + k_1 = 4 \longrightarrow \alpha_c(s) = s^2 + 4s + 8$

$\alpha_2 = 2 + k_2 = 8$

PÓLOS : $s = \frac{-4 \pm \sqrt{-16}}{2} = -2 \pm 2j$

c) $\sigma = 2 \ ; \ \omega_d = 2 \ ; \ \omega_n = 2\sqrt{2} \ ; \ \xi = 0.7$

$\xi = 0.7 \longrightarrow M_p = 5\%$

$t_r = 0.52$ seg ; $t_s = 2.3$ seg ; $t_p = 1.57$ seg

d) $u = -[2 \ \ 6]x + \bar{N}r$

ENTÃO : $x' = \left(\begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [2 \ \ 6] \right) x + 6\bar{N}r$

$y = [1 \ \ 20]\, x$

$\frac{Y(s)}{R(s)} = \begin{pmatrix} 1 & 20 \end{pmatrix} \begin{pmatrix} s+4 & 8 \\ -1 & s \end{pmatrix}^{-1} \begin{pmatrix} \bar{N} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 20 \end{pmatrix} \frac{\begin{pmatrix} s & -8 \\ 1 & s+4 \end{pmatrix}}{s^2+4s+8} \begin{pmatrix} \bar{N} \\ 0 \end{pmatrix}$

$\frac{Y(s)}{R(s)} = \frac{\bar{N}(s+20)}{s^2+4s+8}$, SENDO $\bar{N} = 0.4$ PARA QUE $\lim_{s \to 0} \frac{Y(s)}{R(s)} = 1.0$

③ a) $\alpha_e(s) = (s+4)^2 = s^2 + 8s + 16$

$FCO: \ \ell_1 = 8 \ \in \ \ell_2 = 17 \longrightarrow L = \begin{bmatrix} 8 \\ 17 \end{bmatrix}$ (CHAMADO DE L_{co} NO ITEM (b))

b) FCC:

$$x' = F_{cc}x + G_{cc}u \qquad \xrightarrow{x = Tz} \qquad z' = F_{co}z + G_{co}u$$

$$y = H_{cc}x \qquad \xleftarrow[z = Px]{} \qquad y = H_{co}z$$

FCO:

$K_{cc} = [4 \ \ 53]$

COMPENSADOR NO FCO: $\begin{cases} \hat{z}' = (F_{co} - G_{co}K_{co} - L_{co}H_{co})\hat{z} + \overset{(L_{co})}{L}y \\ u = -K_{co}\hat{z} = -K_{co}T\hat{x} = -K_{cc}\hat{x} \end{cases}$

PRECISAMOS SABER $\underline{\underline{T}}$. CÁLCULO DA TRANSFORMAÇÃO LINEAR:

$F_{cc} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad G_{cc} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad H_{cc} = (0 \ \ 1) \longrightarrow \mathcal{O}_{cc} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$t_2 = \mathcal{O}^{-1}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ ; \quad t_1 = Ft_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

ENTÃO: $T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \ \in \ K_{co} = [5 \ \ 4]$

VOLTANDO AO COMPENSADOR NO FCO:

$$F_{co} - G_{co}K_{co} - L_{co}H_{co} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}[5 \ \ 4] - \begin{bmatrix} 8 \\ 17 \end{bmatrix}[1 \ \ 0] = \begin{bmatrix} -8 & 1 \\ -21 & -4 \end{bmatrix}$$

$$\hat{z}' = \begin{bmatrix} -8 & 1 \\ -21 & -4 \end{bmatrix}\hat{z} + \begin{bmatrix} 8 \\ 17 \end{bmatrix}y$$

$$u = -[5 \ \ 4]\hat{z}$$

c) $$D(s) = -(5 \ \ 4)\begin{pmatrix} s+8 & -1 \\ 21 & s+4 \end{pmatrix}^{-1}\begin{pmatrix} 8 \\ 17 \end{pmatrix} = \frac{-(5 \ \ 4)\begin{pmatrix} s+4 & 1 \\ -21 & s+8 \end{pmatrix}\begin{pmatrix} 8 \\ 17 \end{pmatrix}}{s^2 + 12s + 53}$$

$$= \frac{(-5s - 20 + 84 \quad -5 - 4s - 32)\begin{pmatrix} 8 \\ 17 \end{pmatrix}}{s^2 + 12s + 53} = \frac{-108s - 117}{s^2 + 12s + 53}$$

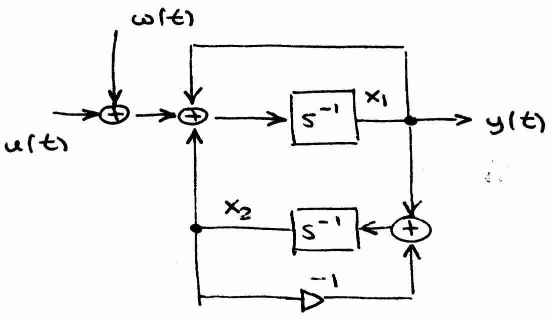
$$\frac{G(s)}{1 - G(s)D(s)} = \frac{\frac{1}{s^2 - 1}}{1 + \frac{108s + 117}{(s^2-1)(s^2 + 12s + 53)}} = \frac{\overbrace{s^2 + 12s + 53}^{\gamma(s), \text{ SE } M = 0}}{s^4 + 12s^3 + 52s^2 + 96s + 64}$$

$$\begin{array}{r} s^4 + 12s^3 + 53s^2 \\ - s^2 - 12s - 53 \\ + 108s + 117 \\ \hline s^4 + 12s^3 + 52s^2 + 96s + 64 \end{array}$$

$$s^4 + 12s^3 + 52s^2 + 96s + 64 = \underbrace{(s^2 + 8s + 16)}_{\alpha_e(s)}\underbrace{(s^2 + 4s + 4)}_{\alpha_c(s) : \ K_{cc} = [4 \ \ 53]}$$

d) $\gamma(s) = s^2 + 8s + 16 = |sI - F_{co} + G_{co}K_{co} + L_{co}H_{co}| \iff M = G_{co}\bar{N} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

4) a) PLANTA:



$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w$$

$$y = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b) CONTROLE INTEGRAL

$$F_i - G_i K = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [K_i \quad K_1 \quad K_2] = \begin{bmatrix} 0 & 1 & 0 \\ -K_i & 1-K_1 & 1-K_2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$|sI - F_i + G_i K| = \begin{vmatrix} s & -1 & 0 \\ K_i & s+K_1-1 & K_2-1 \\ 0 & -1 & s+1 \end{vmatrix} = s^3 + K_1 s^2 + (K_1-1)s + (K_2-1)s + K_i s + K_i$$

ENTÃO: $s^3 + K_1 s^2 + (K_1 + K_2 + K_i - 2)s + K_i = s^3 + 9s^2 + 27s + 27$

$K_1 = 9$
$K_i = 27$ $\longrightarrow$ $K_1 + K_2 + K_i = 29$ $\longrightarrow$ $K_2 = -7$

PORTANTO $K = [K_i \quad K_1 \quad K_2] = [27 \quad 9 \quad -7]$

c)
$$\begin{bmatrix} x_i' \\ x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -27 & -8 & 8 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_i \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} w$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_i \\ x_1 \\ x_2 \end{bmatrix}$$

$$\frac{Y(s)}{W(s)} = (0 \quad 1 \quad 0)\begin{pmatrix} s & -1 & 0 \\ 27 & s+8 & -8 \\ 0 & -1 & s+1 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{s(s+1)}{s^3 + 9s^2 + 27s + 27}$$

$$\frac{Y(s)}{W(s)} = \frac{s(s+1)}{(s+3)^3}$$

d)
$$F = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \begin{matrix} F_{aa} & F_{ab} \\ F_{ba} & F_{bb} \end{matrix}$$

(NOTE QUE $H = [1 \quad 0]$)

$$|sI - F_{ab} + LF_{ab}| = s + 1 + L = \underbrace{s + 6}_{\alpha_e(s)}$$

PORTANTO, $L = 5$

5 a) $x_1' = x_2 = f_1(x_1, x_2, x_3, u)$

$x_2' = x_3 = f_2(x_1, x_2, x_3, u)$

$x_3' = -x_1^3 + u = f_3(x_1, x_2, x_3, u)$

EQUILÍBRIO: $u = u_0 = 1$

$x_{20} = 0$; $x_{30} = 0$; $-x_{10}^3 + 1 = 0 \therefore x_{10}^3 = 1$

$x_{10} \in \mathbb{R} \Rightarrow x_{10} = 1$

b) $\frac{df_1}{dx_1} = 0 \quad \frac{df_1}{dx_2} = 1 \quad \frac{df_1}{dx_3} = 0 \quad \frac{df_1}{du} = 0$

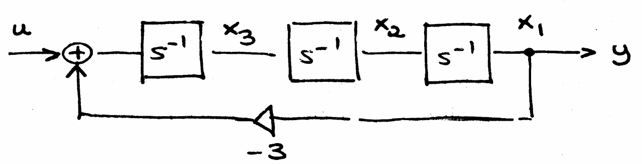
$\frac{df_2}{dx_1} = 0 \quad \frac{df_2}{dx_2} = 0 \quad \frac{df_2}{dx_3} = 1 \quad \frac{df_2}{du} = 0$

$\frac{df_3}{dx_1} = -3x_1^2 \Big|_{x_1 = x_{10}} = -3 \quad \frac{df_3}{dx_2} = 0 \quad \frac{df_3}{dx_3} = 0 \quad \frac{df_3}{du} = 1$

$$x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0]\, x$$

c)



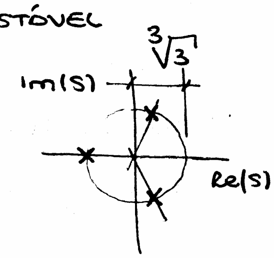
d) $$\mathcal{C} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \det \mathcal{C} = -1 \qquad \text{CONTROLÁVEL}$$

$$\mathcal{O} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \det \mathcal{O} = 1 \qquad \text{OBSERVÁVEL}$$

$$|sI - F| = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 3 & 0 & s \end{vmatrix} = s^3 + 3 = 0 \qquad \text{SISTEMA INSTÁVEL}$$

$s_1 = -\sqrt[3]{3}$; $s_2 = \sqrt[3]{3} \measuredangle 60^\circ$

$s_3 = \sqrt[3]{3} \measuredangle -60^\circ$



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