EEL760 —— CONTROLE LINEAR II —— 26/09/2005

GABARITO DA PROVA PARCIAL #1

PROBLEMA #1:

a) $(s+1)x_1 = u + \alpha x_2$ $\qquad x_1' = -x_1 + \alpha x_2 + u$

$(s+2)x_2 = u + x_1$ $\qquad x_2' = -2x_2 + x_1 + u$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & \alpha \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b) $\mathcal{C} = \begin{bmatrix} 1 & \alpha-1 \\ 1 & -1 \end{bmatrix}$ $\qquad \det \mathcal{C} = 1 - \alpha - 1 = -\alpha = 0$

$\alpha = 0 \implies$ SISTEMA NÃO-CONTROLÁVEL

c) $\mathcal{O} = \begin{bmatrix} 1 & 1 \\ 0 & \alpha-2 \end{bmatrix}$ $\qquad \det \mathcal{O} = \alpha - 2 = 0$

$\alpha = 2 \implies$ SISTEMA NÃO-OBSERVÁVEL

d) $|sI - F| = \begin{vmatrix} s+1 & -\alpha \\ -1 & s+2 \end{vmatrix}$

$= s^2 + 3s + 2 - \alpha = 0 \implies s = \dfrac{-3 \pm \sqrt{9-8+4\alpha}}{2} = \dfrac{-3 \pm \sqrt{4\alpha+1}}{2}$

SISTEMA INSTÁVEL SE PARTE REAL DE UM PÓLO FOR MAIOR QUE ZERO:

$\sqrt{4\alpha+1} > 3 \longrightarrow 4\alpha + 1 > 9 \longrightarrow \alpha > 2$

e) $$G(s) = \frac{[1 \quad 1]\begin{bmatrix} s+2 & \alpha \\ 1 & s+1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2 + 3s + 2 - \alpha} = \frac{2s + 4 + \alpha}{s^2 + 3s + 2 - \alpha}$$

PROBLEMA #2:

a) $$G(s) = \frac{[1 \quad 0]\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2 - 1} = \frac{s}{s^2-1}$$

$$Y(s) = \frac{G(s)}{s} = \frac{1}{(s+1)(s-1)} = \frac{A}{s+1} + \frac{B}{s-1} \qquad A = -\frac{1}{2} \ ; \ B = \frac{1}{2}$$

$$y(t) = \left(-\frac{1}{2}e^{-t} + \frac{1}{2}e^{t}\right)u(t)$$

b) $$H(sI-F)^{-1}x(0) = \frac{[1 \quad 0]\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2-1} = \frac{s}{s^2-1}$$

$$Y(s) = \frac{1}{s^2-1} + \frac{s}{s^2-1} = \frac{s+1}{(s+1)(s-1)} = \frac{1}{s-1} \longrightarrow y(t) = e^{t}u(t)$$

c) $$H(sI-F)^{-1}x(0) = \frac{[1 \quad 0]\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}\begin{bmatrix} 11 \\ 9 \end{bmatrix}}{s^2-1} = \frac{11s+9}{s^2-1}$$

$$Y(s) = \frac{1}{s^2-1} + \frac{11s+9}{s^2-1} = \frac{11s+10}{(s+1)(s-1)} = \frac{A}{s+1} + \frac{B}{s-1} \qquad A = \frac{1}{2} \ ; \ B = \frac{21}{2}$$

$$y(t) = \left(\frac{1}{2}e^{-t} + \frac{21}{2}e^{t}\right)u(t)$$

PROBLEMA #3:

a) $\alpha_c(s) = (s+1+j)(s+1-j) = s^2 + 2s + 2$

$$F - Gk = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}[k_1 \quad k_2] = \cancel{\begin{bmatrix} 1-2k_1 \\ 1- \end{bmatrix}} = \begin{bmatrix} 1-k_1 & 2-k_2 \\ 1-k_1 & -k_2 \end{bmatrix}$$

$$|sI - F + GK| = \begin{vmatrix} s+k_1-1 & k_2-2 \\ k_1-1 & s+k_2 \end{vmatrix} = s^2 + (k_1+k_2-1)s + \cancel{k_1k_2} - \cancel{k_2} - \cancel{k_1k_2} + 2k_1 + \cancel{k_2} - 2$$

$$k_1 + k_2 - 1 = 2$$

$$2k_1 - 2 = 2 \longrightarrow k_1 = 2 \longrightarrow k_2 = 1$$

$$K = [2 \quad 1]$$

$$\alpha_e(s) = (s+2+2j)(s+2-2j) = s^2 + 4s + 8$$

$$F - LH = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} [1 \quad 0] = \begin{bmatrix} 1-\ell_1 & 2 \\ 1-\ell_2 & 0 \end{bmatrix}$$

$$|sI - F + LH| = \begin{vmatrix} s+\ell_1-1 & -2 \\ \ell_2-1 & s \end{vmatrix} = s^2 + (\ell_1-1)s + \cancel{2\ell_1}\, 2\ell_2 - 2$$

$$\ell_1 - 1 = 4$$

$$2\ell_2 - 2 = 8 \longrightarrow \ell_1 = 5 \; ; \; \ell_2 = 5 \; ; \quad L = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

b) $$G(s) = \frac{[1 \quad 0]\begin{bmatrix} s & 2 \\ 1 & s-1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2 - s - 2} = \frac{s+2}{(s+1)(s-2)}$$

D(s) : $$F - GK - LH = \underbrace{\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}[2 \quad 1]}_{\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}}}_{\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}} - \underbrace{\begin{bmatrix} 5 \\ 5 \end{bmatrix}[1 \quad 0]}_{\begin{bmatrix} 5 & 0 \\ 5 & 0 \end{bmatrix}} = \begin{bmatrix} -6 & 1 \\ -6 & -1 \end{bmatrix}$$

$$D(s) = -K(sI - F + GK + LH)^{-1} L = -[2 \quad 1]\begin{bmatrix} s+6 & -1 \\ 6 & s+1 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$D(s) = \frac{-[2 \quad 1]\begin{bmatrix} s+1 & 1 \\ -6 & s+6 \end{bmatrix}\begin{bmatrix} 5 \\ 5 \end{bmatrix}}{s^2 + 7s + 12} = \frac{-(15s + 20)}{s^2 + 7s + 12}$$

c) SE $\bar{N} = 1$ E $M = 0$:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 - G(s)D(s)} = \frac{\dfrac{s+2}{(s+1)(s-2)}}{1 + \dfrac{s+2}{s^2-s-2} \cdot \dfrac{15s+20}{s^2+7s+12}} = \frac{(s+2)(s^2+7s+12)}{(s^2-s-2)(s^2+7s+12) + (s+2)(15s+20)}$$

$$\underbrace{(s^2 - s - 2)(s^2 + 7s + 12)}_{} + \underbrace{(s+2)(15s+20)}_{15s^2 + 30s + 20s + 40} = s^4 + 6s^3 + 18s^2 + 24s + 16$$

$$\begin{array}{r} s^4 + 7s^3 + 12s^2 \qquad\qquad \\ - s^3 - 7s^2 - 12s \qquad \\ - 2s^2 - 14s - 24 \\ \hline s^4 + 6s^3 + 3s^2 - 26s - 24 \end{array}$$

DIVIDINDO O DENOMINADOR POR $\alpha_e(s)$:

$$\begin{array}{r|l} s^4 + 6s^3 + 18s^2 + 24s + 16 & s^2 + 4s + 8 \\ \cline{2-2} -(s^4 + 4s^3 + 8s^2) \qquad\qquad & s^2 + 2s + 2 \\ \cline{1-1} 2s^3 + 10s^2 + 24s \qquad & \\ -(2s^3 + 8s^2 + 16s) \qquad & \\ \cline{1-1} 2s^2 + 8s + 16 & \\ 2s^2 + 8s + 16 & \\ \cline{1-1} 0 & \end{array}$$

ENTÃO : $$\frac{Y(s)}{R(s)} = \frac{(s+2)(s+3)(s+4)}{(s^2+2s+2)(s^2+4s+8)} = \frac{(s+2)(s+3)(s+4)}{\alpha_c(s)\,\alpha_e(s)}$$

GANHO DC : $$\lim_{s \to 0} \frac{Y(s)}{R(s)} = \frac{24}{16} = \frac{3}{2}$$

d) $$\begin{array}{c} \qquad F_{ab} \\ \left[\begin{array}{c|c} 1 & 2 \\ \hline 1 & 0 \end{array}\right] \\ \qquad F_{bb} \end{array} \qquad |sI - F_{bb} + LF_{ab}| = s + 2L = s + 6 \longrightarrow L = 3$$

e) PARA QUE $\gamma(s) = \alpha_e(s) = |sI - F + LH|$, PRECISAMOS QUE $\dfrac{M}{\bar{N}} = G$: $\quad M = \begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix}$

PROBLEMA #4:

a) $x_1' = u - x_1^2 - \sqrt{x_2} = f_1(x_1, x_2, u)$

$x_2' = x_1 = f_2(x_1, x_2, u)$

$f_1(x_{10}, x_{20}, u_0) = 0 \implies u_0 - x_{10}^2 - \sqrt{x_{20}} = 0$

$f_2(x_{10}, x_{20}, u_0) = 0 \implies x_{10} = 0$

$\sqrt{x_{20}} = 2 \longrightarrow x_{20} = 4$

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{x_{10}, x_{20}, u_0} = -2x_{10} = 0$$

$$\left.\frac{\partial f_2}{\partial x_1}\right|_{x_{10}, x_{20}, u_0} = 1 \qquad \left.\frac{\partial f_1}{\partial x_2}\right|_{x_{10}, x_{20}, u_0} = \frac{-1}{2\sqrt{x_{20}}} = -\frac{1}{4} \qquad \left.\frac{\partial f_2}{\partial x_2}\right|_{x_{10}, x_{20}, u_0} = 0$$

$$\left.\frac{\partial f_1}{\partial u}\right|_{x_{10}, x_{20}, u_0} = 1 \qquad \left.\frac{\partial f_2}{\partial u}\right|_{x_{10}, x_{20}, u_0} = 0$$

$$x' = \begin{bmatrix} 0 & -\frac{1}{4} \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = [0 \quad 1] x$$

PÓLOS: $\begin{vmatrix} s & \frac{1}{4} \\ -1 & s \end{vmatrix} = 0 \implies s^2 = -\frac{1}{4} \implies s = \pm \frac{j}{2}$; $\mathcal{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\mathcal{O} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

b) $(s+1) x_1 = u x_1 x_2$

$x_1' = -x_1 + u x_1 x_2 = f_1(x_1, x_2, u)$

$f_1(x_{10}, x_{20}, u_0) = 0 \overset{u_0 = 1}{\implies} x_{10} x_{20} = x_{10}$

$x_{20} = 1 \quad (x_{10} \neq 0)$

$(s+2) x_2 = u x_1 x_2$

$x_2' = -2x_2 + u x_1 x_2 = f_2(x_1, x_2, u)$

$f_2(x_{10}, x_{20}, u_0) = 0 \overset{u_0 = 1}{\implies} x_{10} x_{20} = 2x_{20}$

$x_{10} = 2 \quad (x_{20} \neq 0)$

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{u_0, x_{10}, x_{20}} = u_0 x_{20} - 1 = 0 \qquad \left.\frac{\partial f_1}{\partial x_2}\right|_{x_{10}, x_{20}, u_0} = u_0 x_{10} = 2$$

$$\left.\frac{\partial f_2}{\partial x_1}\right|_{u_0, x_{10}, x_{20}} = u_0 x_{20} = 1 \qquad \left.\frac{\partial f_2}{\partial x_2}\right|_{x_{10}, x_{20}, u_0} = u_0 x_{10} - 2 = 0$$

$$\left.\frac{\partial f_1}{\partial u}\right|_{u_0, x_{10}, x_{20}} = x_{10} x_{20} = 2 \qquad \left.\frac{\partial f_2}{\partial u}\right|_{u_0, x_{10}, x_{20}} = x_{10} x_{20} = 2$$

$$x' = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 2 \end{bmatrix} u$$

$$y = [1 \quad 0] x$$

PÓLOS: $\begin{vmatrix} s & -2 \\ -1 & s \end{vmatrix} = 0 \implies s^2 = 2 \implies s = \pm\sqrt{2}$; $\mathcal{C} = \begin{bmatrix} 2 & 4 \\ 2 & 2 \end{bmatrix}$; $\mathcal{O} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

PROBLEMA #5:

a) PLANTA (DUPLO INTEGRADOR) EM MALHA ABERTA: $x' = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

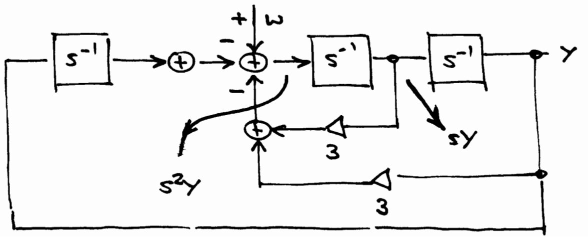
$y = [0 \quad 1] x$

CONTROLE INTEGRAL: $F_i - G_i k = \left[\begin{array}{c|cc} 0 & 0 & 1 \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} [k_1 \quad k_2 \quad k_3] = \begin{bmatrix} 0 & 0 & 1 \\ -k_1 & -k_2 & -k_3 \\ 0 & 1 & 0 \end{bmatrix}$

$\alpha_c(s) = (s+1)^3 = s^3 + 3s^2 + 3s + 1$

$|sI - F_i + G_i k| = \begin{vmatrix} s & 0 & -1 \\ k_1 & s + k_2 & k_3 \\ 0 & -1 & s \end{vmatrix} = s^3 + k_2 s^2 + k_3 s + k_1$: $k_1 = 1$, $k_2 = 3$, $k_3 = 3$

b)



POR INSPEÇÃO: $W - 3sY - 3Y - \frac{Y}{s} = s^2 Y$

$sW = (s^3 + 3s^2 + 3s + 1) Y$

$$\frac{Y}{W} = \frac{s}{s^3 + 3s^2 + 3s + 1}$$