CONTROLE LINEAR II — PROVA PARCIAL #1 — GABARITO

QUESTÃO 1.

a) $Y(s) = H(sI-F)^{-1}G\,.\,\frac{1}{s} + J\,.\,\frac{1}{s}$

$$= \frac{[6 \;\; -2]\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s(s+1)(s+2)} + \frac{1}{s} = \frac{4s+10}{s(s+1)(s+2)} + \frac{1}{s}$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{1}{s}$$

$A = \frac{10}{2}$ $\quad B = \frac{6}{-1}$ $\quad C = \frac{2}{2}$ $\longrightarrow$ $y(t) = (6 - 6e^{-t} + e^{-2t})\,u(t)$

b) $Y(s) = H(sI-F)^{-1}x(0) = \frac{[6 \;\; -2]\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{(s+1)(s+2)} = \frac{4s+10}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$

$A = \frac{6}{1}$ $\quad B = \frac{2}{-1}$

$y(t) = (6e^{-t} - 2e^{-2t})\,u(t)$

c) $\frac{Y(s)}{U(s)} = H(sI-F)^{-1}G + J = \frac{4s+10+s^2+3s+2}{s^2+3s+2} = \frac{s^2+7s+12}{s^2+3s+2}$

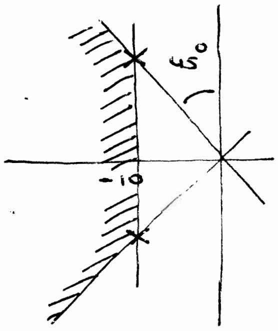
ZEROS. $z = \frac{-7 \pm \sqrt{49-48}}{2} \Rightarrow z_1 = -3\,;\; z_2 = -4.$

QUESTÃO 2:

a) $F = \begin{bmatrix} -5 & -4 \\ 1 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad H = [0 \;\; 1]$

$M_p \leq 5\% \longrightarrow \xi \geq 0.7 \quad (\theta \geq 45^\circ)$

$t_s \leq 0.46\,seg \longrightarrow \sigma \geq \frac{4.6}{0.46} = 10$




ESTE RESULTADO DEPENDE DA ESCOLHA DE F E G.

$s = -10 \pm 10j$

$\alpha_c(s) = (s+10+10j)(s+10-10j) = s^2 + 20s + 200$

$F - GK = \begin{bmatrix} -5-k_1 & -4-k_2 \\ 1 & 0 \end{bmatrix}$ $\quad k_1 + 5 = 20$, $\quad k_2 + 4 = 200$ $\longrightarrow$ $K = [15 \;\; 196]$

b) $x' = (F - GK)x + G\bar{N}r$

$y = Hx$

$$F - GK = \begin{bmatrix} -20 & -200 \\ 1 & 0 \end{bmatrix}$$

ASSUMINDO $\bar{N} = 1$ :

$$\frac{Y(s)}{R(s)} = H(sI - F + GK)^{-1} G = \frac{\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} s & -200 \\ 1 & s+20 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2 + 20s + 200}$$

$$\frac{Y(s)}{R(s)} = \frac{1}{s^2 + 20s + 200}$$

PARA QUE O GANHO DE SEJA UNITÁRIO : $\bar{N} = 200$

EQUAÇÕES DE ESTADO:

$$x' = \begin{bmatrix} -20 & -200 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 200 \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

QUESTÃO 3 :

a)

$$F - LH = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} = \begin{bmatrix} -3-2\ell_1 & 1-\ell_1 \\ -2-2\ell_2 & -\ell_2 \end{bmatrix}$$

$$\begin{vmatrix} s+2\ell_1+3 & \ell_1-1 \\ 2\ell_2+2 & s+\ell_2 \end{vmatrix} = s^2 + (2\ell_1 + \ell_2 + 3)s + 3\ell_2 + \cancel{2\ell_1\ell_2} - \cancel{2\ell_1\ell_2} - 2\ell_1 + 2\ell_2 + 2$$

$$= s^2 + (2\ell_1 + \ell_2 + 3)s + (-2\ell_1 + 5\ell_2 + 2)$$

$$\alpha_e(s) = (s+5)(s+10) = s^2 + 15s + 50$$

$$\begin{cases} 2\ell_1 + \ell_2 = 12 \\ -2\ell_1 + 5\ell_2 = 48 \end{cases} \longrightarrow L = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

b)

$$z = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x = Px$$

$$T = P^{-1} = \frac{\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}}{2}$$

$$T^{-1}FT = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \frac{\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}}{2} = \begin{bmatrix} -8 & 2 \\ -2 & 0 \end{bmatrix} \frac{\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}}{2} = \left[\begin{array}{c|c} -4 & \overset{F_{ab}}{6} \\ \hline -1 & \underset{F_{bb}}{1} \end{array}\right]$$

$$F_{bb} - LF_{ab} = 1 - 6L$$

$s + 6L - 1 = s + 5 \longrightarrow L = 1 \longrightarrow$ ESTE RESULTADO DEPENDE DA ESCOLHA DE P.

QUESTÃO 4.

a) $F - GK = \begin{bmatrix} -2-k_1 & -2-k_2 \\ 1 & 0 \end{bmatrix} \longrightarrow \det(sI - F + GK) = s^2 + (k_1+2)s + k_2 + 2$

$\alpha_c(s) = (s+2)^2 = s^2 + 4s + 4$

$k_1 = 2 \qquad k_2 = 2 \qquad K = [\underbrace{2}_{K_a} \,|\, \underbrace{2}_{K_b}]$

$$\begin{array}{ccc} & F_{aa} & F_{ab} \\ & \left[\begin{array}{c|c} -2 & -2 \\ \hline 1 & 0 \end{array}\right] & \\ F_{ba} & & F_{bb} \end{array}$$

b) $F_{bb} - LF_{ab} = 2L$

$s - 2L = s + 4$

$L = -2$

c) $F_r = F_{bb} - LF_{ab} - (G_b - LG_a)K_b = -8$

$G_r = F_r L + F_{ba} - LF_{aa} - (G_b - LG_a)K_a = 9$

$H_r = -K_b = -2$

$J_r = -K_a - K_b L = 2$

$$\begin{array}{c} \left[\begin{array}{c} 1 \\ \hline 0 \end{array}\right] \begin{array}{l} G_a \\ G_b \end{array} \end{array}$$

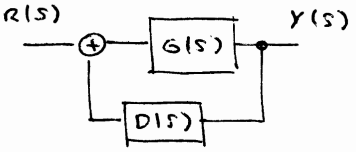
$$\begin{cases} x_c' = -8x_c + 9y \\ u = -2x_c + 2y \end{cases}$$

$D(s) = H_r(sI - F_r)^{-1}G_r + J_r = \dfrac{-2 \times 9}{s+8} + 2 = \dfrac{2s-2}{s+8}$

d) $G(s) = \dfrac{s}{s^2+2s+2}$ (POR INSPEÇÃO)

ALTERNATIVA: $G(s) = \dfrac{[1 \;\; 0]\begin{bmatrix} s & -2 \\ 1 & s+2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2+2s+2} = \dfrac{s}{s^2+2s+2}$

e)




$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 - G(s)D(s)} = \frac{\dfrac{s}{s^2+2s+2}}{1 - \dfrac{s(2s-2)}{(s^2+2s+2)(s+8)}} = \frac{s(s+8)}{s^3+2s^2+2s+8s^2+16s+16-2s^2+2s}$$

$$\frac{Y(s)}{R(s)} = \frac{(s+8)s}{s^3+8s^2+20s+16}$$

NOTE QUE $s = -2$ E $s = -4$ SÃO RAÍZES DO DENOMINADOR.

$(s+2)(s+2)(s+4) = (s^2+4s+4)(s+4) = s^3 + 4s^2 + 4s + 4s^2 + 16s + 16$

$= s^3 + 8s^2 + 20s + 16$

QUESTÃO 5:

a)
$$\dot{x} = -x + u - r + w$$
$$y = x$$

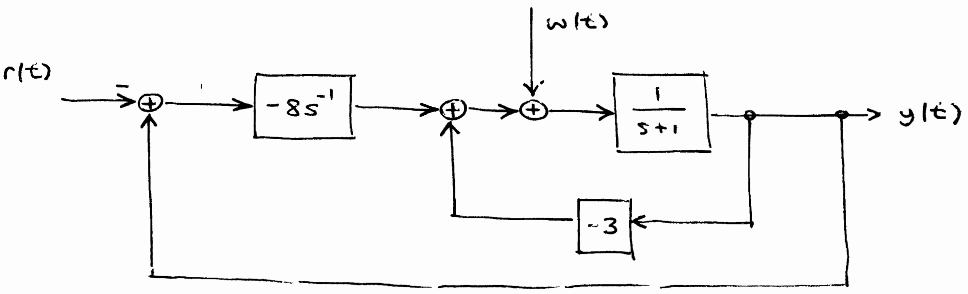
$$\begin{bmatrix} \dot{x}_i \\ \dot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & H \\ 0 & F \end{bmatrix}}_{\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}} \begin{bmatrix} x_i \\ x \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ G \end{bmatrix}}_{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} u - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ G_1 \end{bmatrix} w$$

$$F_i - G_i K_i = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1 \;\; k_2] = \begin{bmatrix} 0 & 1 \\ -k_1 & -1-k_2 \end{bmatrix}$$

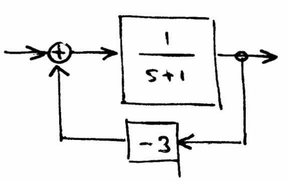
$$\begin{vmatrix} s & -1 \\ k_1 & s+k_2+1 \end{vmatrix} = s^2 + (k_2+1)s + k_1$$

$$\alpha_c(s) = (s+2+2j)(s+2-2j) = s^2 + 4s + 8 \longrightarrow K = [8 \;\; 3]$$

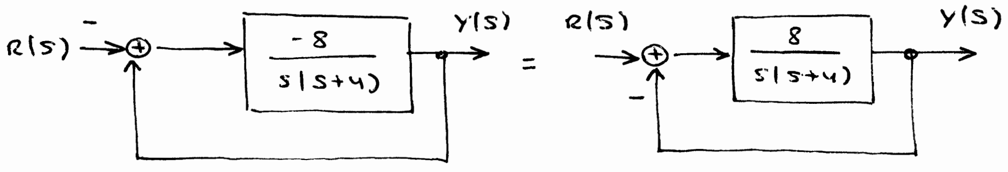
b)




c) $\dfrac{Y(s)}{R(s)}$




$$= \frac{\frac{1}{s+1}}{1 + \frac{3}{s+1}} = \frac{1}{s+4} \qquad (w(s) = 0)$$




$$\frac{Y(s)}{R(s)} = \frac{\frac{8}{s(s+4)}}{1 + \frac{8}{s(s+4)}} = \frac{8}{s^2+4s+8}$$

$\dfrac{Y(s)}{W(s)}$ : FAZENDO $R(s) = 0$, NOTE QUE:

$$Y(s) = \frac{1}{s+1}\left( W(s) - 3Y(s) - \frac{8}{s}Y(s) \right)$$

$$\underbrace{\left( s+1+3+\frac{8}{s} \right)}_{\frac{s^2+4s+8}{s}} Y(s) = W(s) \implies \frac{Y(s)}{W(s)} = \frac{s}{s^2+4s+8}$$

QUESTÃO 6:

$$x_1' = \left(u - \frac{1}{2}x_1 - x_2\right)^2 - 1 = f_1(x_1, x_2, u)$$

$$x_2' = x_1 = f_2(x_1, x_2, u)$$

a) EQUILÍBRIO PARA $u_0 = 1$ E $x_2 > 0$:

$x_2' = 0 \implies x_{10} = 0$

$x_1' = 0 \implies \left(u_0 - \frac{1}{2}x_{10} - x_{20}\right)^2 - 1 = 0$

$(1 - x_{20})^2 = 1$

DUAS SOLUÇÕES: $x_{20} = 0$

$x_{20} = 2 \longrightarrow$ USAR ESTA SOLUÇÃO: $u_0 = 1$, $x_{10} = 0$, $x_{20} = 2$

$$\left.\frac{\partial f_1}{\partial x_1}\right|_{u_0, x_{10}, x_{20}} = \left. -2\left(u - \frac{1}{2}x_1 - x_2\right)\frac{1}{2}\right|_{u_0, x_{10}, x_{20}} = 1$$

$$\left.\frac{\partial f_1}{\partial x_2}\right|_{u_0, x_{10}, x_{20}} = \left. -2\left(u - \frac{1}{2}x_1 - x_2\right)\right|_{u_0, x_{10}, x_{20}} = 2$$

$$\frac{\partial f_2}{\partial x_1} = 1$$

$$\frac{\partial f_2}{\partial x_2} = 0$$

$$\left.\frac{\partial f_1}{\partial u}\right|_{u_0, x_{10}, x_{20}} = \left. 2\left(u - \frac{1}{2}x_1 - x_2\right)\right|_{u_0, x_{10}, x_{20}} = -2$$

$$\frac{\partial f_2}{\partial u} = 0$$

SISTEMA LINEARIZADO SOBRE O PONTO DE EQUILÍBRIO: $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix} u$

$$y = [2 \quad b_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

ESTABILIDADE: $\begin{bmatrix} s-1 & -2 \\ -1 & s \end{bmatrix} \longrightarrow s^2 - s - 2 = 0 \qquad s = \frac{1 \pm \sqrt{1+8}}{2} = -1$ OU $+2$

SISTEMA INSTÁVEL

CONTROLABILIDADE: $\mathcal{C} = \begin{bmatrix} -2 & -2 \\ 0 & -2 \end{bmatrix} \longrightarrow$ SISTEMA CONTROLÁVEL

OBSERVABILIDADE: $\mathcal{O} = \begin{bmatrix} 2 & b_2 \\ b_2+2 & 4 \end{bmatrix} \longrightarrow \det \mathcal{O} = -b_2^2 - 2b_2 + 8 \neq 0$

$b_2^2 + 2b_2 - 8 \neq 0$

$b_2 \neq \frac{-2 \pm \sqrt{4+32}}{2}$

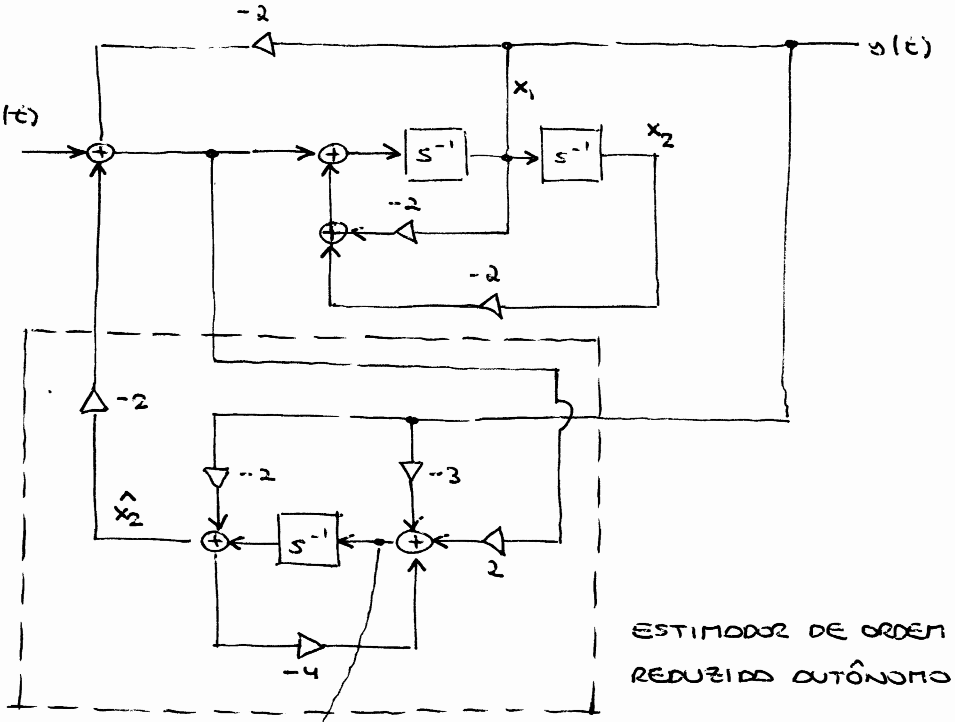
SISTEMA OBSERVÁVEL SE $b_2 \neq -4$ E $b_2 \neq 2$.

QUESTÃO 7:

$F_{ba} - LF_{aa} = 1 + 2L = -3$

$F_{bb} - LF_{ab} = -4$

$G_b - LF_{ab} = 2$




ESTIMADOR DE ORDEM REDUZIDO AUTÔNOMO

$$x_1' = -2x_1 - 2x_2 - 2x_1 - 2\hat{x}_2 + r = -4x_1 - 2x_2 - 2\hat{x}_2 + r$$

$$x_2' = x_1$$

$$\hat{x}_2' + 2y' = -3y + 2\left(-2x_1 - 2\hat{x}_2 + r\right) - 4\hat{x}_2$$

$$= -3x_1 + 2\left(-2x_1 - 2\hat{x}_2 + r\right) - 4\hat{x}_2 = -7x_1 - 8\hat{x}_2 + 2r$$

$$\hat{x}_2' = -7x_1 - 8\hat{x}_2 + 2r - 2\underbrace{\left(-4x_1 - 2x_2 - 2\hat{x}_2 + r\right)}_{y'}$$

ENTÃO $\hat{x}_2' = x_1 + 4x_2 - 4\hat{x}_2$

$$\begin{bmatrix} x_1' \\ x_2' \\ \hat{x}_2' \end{bmatrix} = \begin{bmatrix} -4 & -2 & -2 \\ 1 & 0 & 0 \\ 1 & 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \hat{x}_2 \end{bmatrix}$$

$$\frac{Y(s)}{R(s)} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s+4 & 2 & 2 \\ -1 & s & 0 \\ -1 & -4 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{\begin{vmatrix} s & 0 \\ 0 & s+4 \end{vmatrix}}{s^3 + 8s^2 + 16s + 8 + 2s + 2s + 8}$$

$$= \frac{s\,\cancel{(s+4)}}{(s+2)(s+2)\cancel{(s+4)}} = \frac{s}{s^2 + 4s + 4}$$

$\longrightarrow$ NOTE QUE O PÓLO PROVENIENTE DE $\alpha_e(s)$ FOI CANCELADO.