Gabarito Lista #4 — Controle II

1 a) $D(z) = D(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}}$ $\quad (T=1)$

$$D(z) = \frac{1}{\left(\frac{2(z-1)}{(z+1)}\right)^2} = \frac{z^2+2z+1}{4(z^2-2z+1)} = \frac{0.25(z^2+2z+1)}{(z^2-2z+1)}$$

b) $\frac{Y(z)}{U(z)} = \frac{1}{s^2}$

$F = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad G = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad H = [0 \;\; 1] \quad J = 0$

$e^{Ft} = \mathcal{L}^{-1}\left((sI-F)^{-1}\right)$ #

$$(sI-F)^{-1} = \begin{pmatrix} s & 0 \\ -1 & s \end{pmatrix}^{-1} = \frac{\begin{pmatrix} s & 0 \\ 1 & s \end{pmatrix}}{s^2} = \begin{pmatrix} \frac{1}{s} & 0 \\ \frac{1}{s^2} & \frac{1}{s} \end{pmatrix}$$

$$e^{Ft} = \begin{bmatrix} u(t) & 0 \\ tu(t) & u(t) \end{bmatrix} \longrightarrow \phi = e^{FT} = \begin{bmatrix} 1 & 0 \\ T & 1 \end{bmatrix}$$

$$\Gamma = \int_0^T e^{Ft} G\,dt = \int_0^T \begin{bmatrix} 1 \\ t \end{bmatrix} dt = \begin{bmatrix} T \\ T^2/2 \end{bmatrix}$$

$H = [0 \;\; 1]$

$$D(z) = [0 \;\; 1] \begin{bmatrix} z-1 & 0 \\ -T & z-1 \end{bmatrix}^{-1} \begin{bmatrix} T \\ T^2/2 \end{bmatrix} \quad \leftarrow \left(H(zI-\phi)^{-1}\Gamma \right)$$

$$= \frac{[0 \;\; 1]\begin{bmatrix} z-1 & 0 \\ T & z-1 \end{bmatrix}}{(z-1)^2} \begin{bmatrix} T \\ T^2/2 \end{bmatrix} = \frac{[T \;\; z-1]\begin{bmatrix} T \\ \frac{T^2}{2} \end{bmatrix}}{(z-1)^2} = \frac{T^2 + z\frac{T^2}{2} - \frac{T^2}{2}}{(z-1)^2}$$

$$D(z) = \frac{\frac{T^2}{2}(z+1)}{z^2-2z+1} \,. \quad \text{se } T=1 \longrightarrow D(z) = \frac{0.5(z+1)}{z^2-2z+1}$$

2 a) $$H(z) = H(zI-\phi)^{-1}\Gamma = [0 \;\; 1]\begin{bmatrix} z-0.2 & 0 \\ 0 & z+0.5 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{[0 \;\; 1]\begin{bmatrix} z+0.5 & 0 \\ 0 & z-0.2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}}{z^2+0.3z-0.1}$$

$$H(z) = \frac{2(z-0.2)}{z^2+0.3z-0.1} = \boxed{\frac{2}{z+0.5} = \frac{Y(z)}{U(z)}} \longrightarrow \text{cancelamento pólo/zero}$$

b) $$U(z) = \frac{z}{z-1} \longrightarrow Y(z) = \frac{2z(z-0.2)}{(z-1)(z^2+0.3z-0.1)} \# = \frac{2z\cancel{(z-0.2)}}{(z-1)\cancel{(z-0.2)}(z+0.5)} = \frac{2z}{(z-1)(z+0.5)}$$

$$\frac{Y(z)}{z} = \frac{A}{z-1} + \cancel{\frac{B}{z-0.2}} + \frac{C}{z+0.5} = \frac{4/3}{z-1} - \frac{4/3}{z+0.5}$$

$$A = \frac{2}{(z+0.5)}\Big|_{z=1} = \frac{4}{3} \qquad C = \frac{2}{z-1}\Big|_{z=-0.5} = \frac{-4}{3}$$

$$Y(z) = \frac{\frac{4}{3}z}{z-1} - \frac{\frac{4}{3}z}{z+0.5} \longrightarrow y(k) = \left(\frac{4}{3}\left(1-(-0.5)^k\right)\right)u(k)$$

$$\boxed{\left(\frac{4}{3}\left(1-(-0.5)^k\right)\right)u(k)}$$

c) $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

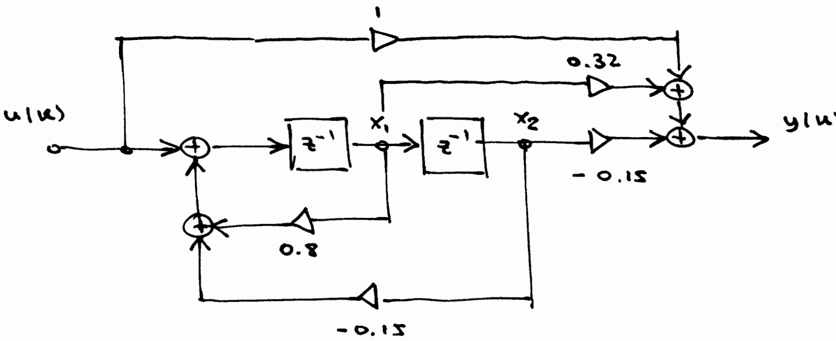
$$Y(z) = Hz(zI - \Phi)^{-1}x(0) = \frac{[0 \;\; 1]\, z \begin{bmatrix} z+0.5 & 0 \\ 0 & z-0.2 \end{bmatrix}}{(z+0.5)(z-0.2)} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{z\cancel{(z-0.2)}}{(z+0.5)\cancel{(z-0.2)}} = \frac{z}{z+0.5}$$

$$Y(z) = \frac{z}{z+0.5} \longrightarrow \boxed{y(k) = (-0.5)^k u(k)}$$

(3) a) $G(z) = 1 + \dfrac{0.32z - 0.15}{z^2 - 0.8z + 0.15}$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.8 & -0.15 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0.32 & -0.15 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + u(k)$$



OBS.: o método de expansão em frações parciais é mais rápido do que a diagonalização do sistema, ~~porque~~ já que não estamos interessados em T neste exercício.

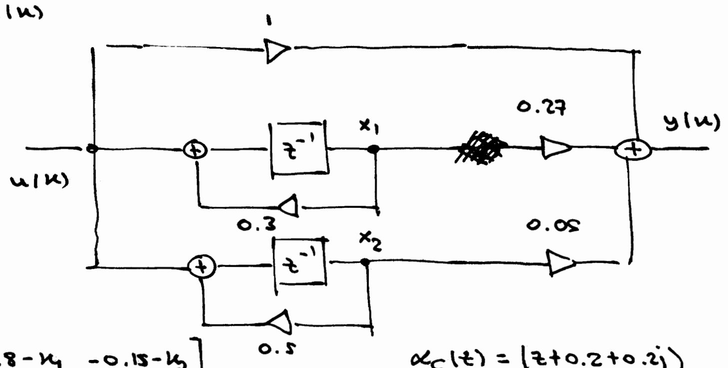
b) $$G(z) = 1 + \frac{0.32z - 0.15}{(z-0.3)(z-0.5)} = 1 + \frac{A}{z-0.3} + \frac{B}{z-0.5} = 1 + \frac{0.27}{z-0.3} + \frac{0.05}{z-0.5}$$

$$A = \frac{0.32 \times 0.3 - 0.15}{-0.2} = 0.27$$

$$B = \frac{0.32 \times 0.5 - 0.15}{0.2} = 0.05$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+2) \end{bmatrix} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0.27 & 0.05 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + u(k)$$



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c) $$\Phi - \Gamma K = \begin{bmatrix} 0.8 & -0.15 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [K_1 \;\; K_2] = \begin{bmatrix} 0.8-K_1 & -0.15-K_2 \\ 1 & 0 \end{bmatrix}$$

$$\alpha_c(z) = (z+0.2+0.2j)(z+0.2-0.2j) = z^2 + 0.4z + 0.08$$

$$|zI - \Phi + \Gamma K| = \begin{vmatrix} z-0.8+K_1 & 0.15+K_2 \\ -1 & z \end{vmatrix} = z^2 + (-0.8+K_1)z + 0.15 + K_2 = z^2 + 0.4z + 0.08$$

$$K_1 = 1.2 \;,\; K_2 = -0.07$$

$$\boxed{K = [1.2 \;\; -0.07]}$$