Gabarito Lista #8 — Controle II

1) a) Pólos: $s_1 = -2$, $s_2 = -3$ ⟶ $z_1 = e^{-2T}$, $z_2 = e^{-3T}$

Zeros: $s_3 = -1$, $s_4 = -4$ ⟶ $z_3 = e^{-T}$, $z_4 = e^{-4T}$

$$D(z) = \frac{(z - e^{-T})(z - e^{-4T})}{(z - e^{-2T})(z - e^{-3T})} \cdot K$$

$$\lim_{s \to 0} D(s)$$

$$\lim_{z \to 1} D(z) = \frac{(1-e^{-T})(1-e^{-4T})}{(1-e^{-2T})(1-e^{-3T})} K = \frac{4}{6} \longrightarrow K = \frac{(1-e^{-2T})(1-e^{-3T})}{(1-e^{-T})(1-e^{-4T})} \cdot \frac{4}{6}$$

Então: $$D(z) = \frac{4}{6} \frac{(1-e^{-2T})(1-e^{-3T})}{(1-e^{-T})(1-e^{-4T})} \frac{(z^2 - z(e^{-T}+e^{-4T}) + e^{-5T})}{(z^2 - z(e^{-2T}+e^{-3T}) + e^{-5T})}$$

b) $$D(z) = D(s)\Big|_{s = \frac{z-1}{T}} = \frac{\left(\frac{z-1}{T}+1\right)\left(\frac{z-1}{T}+4\right)}{\left(\frac{z-1}{T}+2\right)\left(\frac{z-1}{T}+3\right)} = \frac{(z-1+T)(z-1+4T)}{(z-1+2T)(z-1+3T)}$$

$$D(z) = \frac{z^2 + (\cancel{4}^{5}T - 2)z + 1 - 5T + 4T^2}{z^2 + (5T-2)z + 1 - 5T + 6T^2}$$

c) $$D(z) = D(s)\Big|_{s = \frac{z-1}{Tz}} = \frac{\left(\frac{z-1}{Tz}+1\right)\left(\frac{z-1}{Tz}+4\right)}{\left(\frac{z-1}{Tz}+2\right)\left(\frac{z-1}{Tz}+3\right)} = \frac{((T+1)z-1)((4T+1)z-1)}{((2T+1)z-1)((3T+1)z-1)}$$

$$D(z) = \frac{(4T^2 + 5T + 1)z^2 - (5T+2)z + 1}{(6T^2 + 5T + 1)z^2 - (5T+2)z + 1}$$

d) $$D(z) = D(s)\Big|_{s = \frac{2}{T}\frac{(z-1)}{(z+1)}} = \frac{\left(\frac{2}{T}\frac{(z-1)}{(z+1)}+1\right)\left(\frac{2}{T}\frac{(z-1)}{(z+1)}+4\right)}{\left(\frac{2}{T}\frac{(z-1)}{(z+1)}+2\right)\left(\frac{2}{T}\frac{(z-1)}{(z+1)}+3\right)} = \frac{((T+2)z + T-2)((4T+2)z + 4T-2)}{((2T+2)z + 2T-2)((3T+2)z + 3T-2)}$$

$$D(z) = \frac{(4T^2 + 10T + 4)z^2 + (\cancel{4}^{8}T^2 - 8)z + 4T^2 - 10T + 4}{(6T^2 + 10T + 4)z^2 + (12T^2 - 8)z + 6T^2 - 10T + 4}$$

e) $$\frac{D(s)}{s} = \frac{(s+1)(s+4)}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3} = \frac{2/3}{s} + \frac{1}{s+2} - \frac{2/3}{s+3}$$

$A = \frac{4}{6}$ $\quad B = \frac{-2}{-2} = 1$ $\quad C = \frac{-2}{3}$

$A = \frac{2}{3}$

$$\mathcal{L}^{-1}\left(\frac{\cancel{X}D(s)}{s}\right) = \left(\frac{2}{3} + e^{-2t} - \frac{2}{3}e^{-3t}\right)u(t) = u(t) \longleftarrow \text{sinal de controle}$$

Degrau unitário ⟶ diferentes! (infelizmente com a mesma letra).

$$u(k) = \left(\frac{2}{3} + e^{-2Tk} - \frac{2}{3}e^{-3Tk}\right)u(k)$$

$$U(z) = \frac{2}{3}\cdot\frac{z}{z-1} + \frac{z}{z-e^{-2T}} - \frac{2}{3}\cdot\frac{z}{z-e^{-3T}}$$

$$(z-1)(z-e^{-2T})$$

$$D(z) = \frac{z-1}{z}U(z) = \frac{2}{3} + \frac{z-1}{z-e^{-2T}} - \frac{2}{3}\frac{z-1}{z-e^{-3T}} = \frac{\frac{2}{3}\left(z^2 - (e^{-2T}+e^{-3T})z + e^{-5T}\right) + (z-1)(z-e^{-3T}) - \frac{2}{3}}{(z^2 - (e^{-2T}+e^{-3T})z + e^{-5T})}$$

$$D(z) = \frac{\frac{2}{3}\left(z^2-(e^{-2T}+e^{-3T})z+e^{-5T}\right) + z^2-(1+e^{-3T})\cancel{z}+e^{-3T} - \frac{2}{3}\left(z^2-(1+e^{-2T})\cancel{z}+e^{-2T}\right)}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}}$$

$$D(z) = \frac{z^2+\left(\frac{-1}{3}-\frac{5}{3}e^{-3T}\right)z+\left(\frac{2e^{-5T}}{3}+e^{-3T}-\frac{2}{3}e^{-2T}\right)}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}}$$

f) $D(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)} = \cancel{\frac{A}{s+2}+\frac{B}{s+3}}\ \cancel{-\frac{2}{s+2}+\frac{2}{s+3}} = \frac{s^2+5s+4}{s^2+5s+6} = 1-\frac{2}{s^2+5s+6} =$

$1+\frac{A}{s+2}+\frac{B}{s+3}$

$A=\frac{-2}{1}=-2$ $\quad B=\frac{-2}{-1}=2$

$A=\frac{-2}{1}=-2$

$B=\frac{-2}{-1}=2$

$$\begin{cases} x' = Fx + G\cancel{x}\,y \\ u\,\cancel{y} = Hx + J\cancel{x}\,y \end{cases}, \quad \text{ONDE} \quad F=\begin{bmatrix}-2 & 0\\ 0 & -3\end{bmatrix},\ G=\begin{bmatrix}1\\ \cancel{0}\,1\end{bmatrix},\ H=[-2\ \ 2],\ J=1$$

(compensador)

$$\Phi = e^{FT} = \begin{bmatrix} e^{-2T} & 0 \\ 0 & e^{-3T}\end{bmatrix}$$

$$\Gamma = \int_0^T e^{F\sigma}G\,d\sigma = \int_0^T \begin{bmatrix} e^{-2\sigma}\\ e^{-3\sigma}\end{bmatrix} d\sigma = \left.\begin{bmatrix} -\frac{1}{2}e^{-2\sigma} \\ -\frac{1}{3}e^{-3\sigma}\end{bmatrix}\right|_0^T = \begin{bmatrix} \frac{1}{2}-\frac{1}{2}e^{-2T} \\ \frac{1}{3}-\frac{1}{3}e^{-3T}\end{bmatrix}$$

$$D(z) = H(zI-\Phi)^{-1}G + J$$

$$(zI-\Phi)^{-1} = \begin{bmatrix} z-e^{-2T} & 0 \\ 0 & z-e^{-3T}\end{bmatrix}^{-1} = \frac{\begin{bmatrix} z-e^{-3T} & 0 \\ 0 & z-e^{-2T}\end{bmatrix}}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}}$$

ENTÃO:

$$D(z) = \frac{[-2\ \ 2]\begin{bmatrix} z-e^{-3T} & 0 \\ 0 & z-e^{-2T}\end{bmatrix}\begin{bmatrix}\frac{1}{2}(1-e^{-2T}) \\ \frac{1}{3}(1-e^{-3T})\end{bmatrix}}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}} + J = \frac{-(1-e^{-2T})(z-e^{-3T})+\frac{2}{3}(1-e^{-3T})(z-e^{-2T})}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}} + 1$$

$$D(z) = \frac{z^2+\left(-1+\cancel{e^{-2T}}+\frac{2}{3}-\frac{2}{3}e^{-3T}-\cancel{e^{-2T}}-e^{-3T}\right)z+\left(e^{-3T}-\cancel{e^{-5T}}-\frac{2}{3}e^{-2T}+\frac{2}{3}e^{-5T}+\cancel{e^{-5T}}\right)}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}}$$

$$D(z) = \frac{z^2+\left(\frac{-1}{3}-\frac{5}{3}e^{-3T}\right)z+\left(\frac{2}{3}e^{-5T}+e^{-3T}-\frac{2}{3}e^{-2T}\right)}{z^2-(e^{-2T}+e^{-3T})z+e^{-5T}}$$

[(ok): igual ao item (2e)]

2 na página seguinte...

(2) $D(s) = \dfrac{1}{s + \cancel{\Omega_c}}$ → FREQ. DE CORTE Ω_c

$$D(z) = D(s)\Big|_{s = \frac{2}{T}\frac{z-1}{z+1}} = \frac{1}{\frac{2}{T}\frac{z-1}{z+1} + \Omega_c}$$

ISTO JÁ FOI FEITO EM AULA

$$D(e^{jw}) = \frac{1}{\frac{2}{T}\frac{e^{jw}-1}{e^{jw}+1} + \Omega_c} = \frac{1}{j\frac{2}{T}\tan\left(\frac{w}{2}\right) + \Omega_c} \quad (1)$$

$$\frac{e^{jw}-1}{e^{jw}+1} = \frac{e^{+\frac{jw}{2}} - e^{-\frac{jw}{2}}}{e^{\frac{jw}{2}} + e^{-\frac{jw}{2}}} = j\,\frac{\sin\left(\frac{w}{2}\right)}{\cos\left(\frac{w}{2}\right)} = j\tan\left(\frac{w}{2}\right)$$

$$\boxed{\Omega_c = \frac{2}{T}\tan\left(\frac{w_c}{2}\right)}$$

FREQÜÊNCIA DE CORTE w_c → A PARTIR DE (1): $\frac{2}{T}\tan\left(\frac{w_c}{2}\right) = \Omega_c$

$\pi/3$

$$\Omega_c = \frac{2}{T}\tan\left(\frac{\pi}{6}\right)$$

$$\Omega_c = \frac{200\sqrt{3}}{3} = 115.5 \text{ rad/seg}$$

115.5 rad/seg $\qquad$ $\pi/3$ rad

$\boxed{\text{ENTÃO, } \Omega_c = \cancel{X} \text{ PARA QUE } w_c = \cancel{0.1}. \text{ (PRÉ-WARPING).}}$

VERIFICAÇÃO DAS FREQÜÊNCIAS DE CORTE (OPCIONAL).

OBS.: $D(s) = \dfrac{1}{s + 115.5}$ → $D(z) = \dfrac{1}{\frac{2}{T}\frac{z-1}{z+1} + 115.5} = \dfrac{Tz + T}{(115.5T + 2)z + (115.5T - 2)}$

→ $D(z) = \dfrac{0.01z + 0.01}{3.155z - 0.845}$

VERIFICANDO: $\lim_{z\to 1} D(z) = \dfrac{0.02}{2.31} = 8.658\times 10^{-3} = G_0$

$$D(z)\Big|_{z = e^{j\frac{\pi}{3}}} = ?$$

$$= \frac{0.01e^{j\frac{\pi}{3}} + 0.01}{3.155e^{j\frac{\pi}{3}} - 0.845}$$

$$= (4.33 - 4.33j)\times 10^{-3} = a_1$$

NOTE QUE $|a_1| = 6.12\times 10^{-3} = \frac{G_0}{\sqrt{2}}$, ENTÃO A FREQÜÊNCIA DE CORTE É, DE FATO, $w_c = \frac{\pi}{3}$ rad.

(3) $u(t) = Ae^{-\alpha t}u(t)$

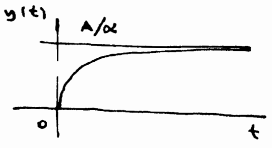
$U(s) = \dfrac{A}{s+\alpha}$

INTEGRADOR: $U(s) \to \boxed{s^{-1}} \to Y(s)$

$$Y(s) = \frac{A}{s(s+\alpha)} = \frac{B}{s} + \frac{C}{s+\alpha} = \frac{A/\alpha}{s} - \frac{A/\alpha}{s+\alpha}$$

$B = \dfrac{A}{\alpha}$ $\qquad$ $C = \dfrac{A}{-\alpha}$

$$y(t) = \left(\frac{A}{\alpha} - \frac{A}{\alpha}e^{-\alpha t}\right)u(t) = \frac{A}{\alpha}\left(1 - e^{-\alpha t}\right)u(t)$$



$u(kT) = A\left(e^{-\alpha T}\right)^k u(kT)$ => $u(k) = Ae^{-\alpha T k}u(k)$ $\qquad$ $U(z) = \dfrac{Az}{z - e^{-\alpha T}}$

• INTEGRADOR RETANGULAR I (FORWARD EULER): $U(z) \longrightarrow \boxed{\dfrac{T}{z-1}} \longrightarrow Y(z)$

$$\frac{Y(z)}{U(z)} = \frac{T}{z-1}$$

$$y(k) = y(k-1) + Tu(k-1) \checkmark$$

$$Y(z) = \frac{Az}{z-e^{-\alpha T}} \cdot \frac{T}{z-1}$$

$$\frac{Y(z)}{z} = \frac{AT}{(z-e^{-\alpha T})(z-1)} = \frac{B}{z-e^{-\alpha T}} + \frac{C}{z-1}$$

$$B = \frac{AT}{e^{-\alpha T}-1} = -\gamma \qquad C = \frac{AT}{1-e^{-\alpha T}} = \gamma$$

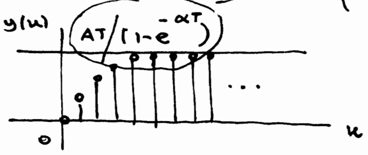
$$Y(z) = \frac{-\gamma z}{z-e^{-\alpha T}} + \frac{\gamma z}{z-1}$$

$$y(k) = \left(\gamma - \gamma e^{-\alpha T k}\right)u(k) = \gamma\left(1-e^{-\alpha T k}\right)u(k)$$

$$\left(\lim_{T \to 0} \frac{AT}{(1-e^{-\alpha T})} = \frac{A}{\alpha}\right)$$

(L'HOPITAL)

$$y(k) = \frac{AT}{1-e^{-\alpha T}}\left(1-e^{-\alpha T k}\right)u(k)$$



• INTEGRADOR RETANGULAR II (BACKWARD EULER): $U(z) \longrightarrow \boxed{\dfrac{Tz}{z-1}} \longrightarrow Y(z)$

$$\frac{Y(z)}{U(z)} = \frac{Tz}{z-1}$$

$$y(k) = y(k-1) + Tu(k) \checkmark$$

$$Y(z) = \frac{Az}{z-e^{-\alpha T}} \cdot \frac{Tz}{z-1}$$

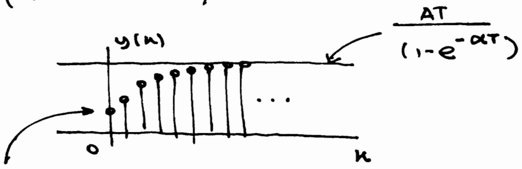
$$\frac{Y(z)}{z} = \frac{ATz}{(z-e^{-\alpha T})(z-1)} = \frac{B}{(z-e^{-\alpha T})} + \frac{C}{z-1}$$

$$B = \frac{ATe^{-\alpha T}}{e^{-\alpha T}-1} = -\gamma e^{-\alpha T} \qquad C = \frac{AT}{1-e^{-\alpha T}} = \gamma$$

$$Y(z) = \frac{-\gamma e^{-\alpha T} z}{(z-e^{-\alpha T})} + \frac{\gamma z}{z-1}$$

$$y(k) = \left(\gamma - \gamma e^{-\alpha T} e^{-\alpha T k}\right)u(k) = \gamma\left(1-e^{-\alpha T(k+1)}\right)u(k)$$

$$y(k) = \frac{AT}{1-e^{-\alpha T}}\left(1-e^{-\alpha T(k+1)}\right)u(k)$$



RESULTADO AVANÇADO DE 1 AMOSTRA.

• INTEGRADOR TRAPEZOIDAL (TUSTIN): $U(z) \longrightarrow \boxed{\dfrac{T}{2}\left(\dfrac{(z+1)}{(z-1)}\right)} \longrightarrow Y(z)$

$$\frac{Y(z)}{U(z)} = \frac{T}{2}\frac{(z+1)}{(z-1)}$$

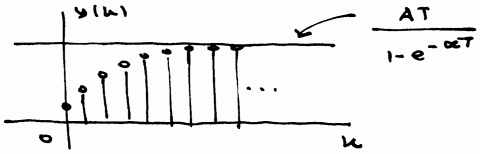
$$y(k) = y(k-1) + \frac{T}{2}\left(u(k-1)+u(k)\right) \checkmark$$

$$Y(z) = \frac{Az}{z-e^{-\alpha T}} \cdot \frac{T}{2}\,\frac{z+1}{z-1}$$

NOTE QUE
$$Y(z) = \frac{1}{2}\left(\overbrace{\frac{ATz^2}{(z-e^{-\alpha T})(z-1)}}^{Y_{\text{BACKWARD EULER}}(z)} + \overbrace{\frac{ATz}{(z-e^{-\alpha T})(z-1)}}^{Y_{\text{FORWARD EULER}}(z)}\right)$$

ENTÃO:
$$y(k) = \frac{1}{2}\left(y_{\text{BACKWARD EULER}}(k) + y_{\text{FORWARD EULER}}(k)\right)$$

$$y(k) = \frac{AT}{1-e^{-\alpha T}}\left(1 - \frac{1}{2}\left(e^{-\alpha T k} + e^{-\alpha T(k+1)}\right)\right)u(k)$$



→ OBTÉM-SE A MÉDIA DOS RESULTADOS DOS OUTROS DOIS MÉTODOS.

(4) a) $F = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}$ $G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $H = [1 \quad 1]$

$$e^{Ft} = \mathcal{L}^{-1}\left[(sI-F)^{-1}\right]$$

$$(sI-F)^{-1} = \begin{bmatrix} s+2 & 2 \\ -1 & s \end{bmatrix}^{-1} = \frac{\begin{bmatrix} s & -2 \\ 1 & s+2 \end{bmatrix}}{s^2+2s+2} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

OBS.: $A(s) = \dfrac{s+1}{s^2+2s+2} \longleftrightarrow a(t) = e^{-t}\cos t\, u(t)$

$B(s) = \dfrac{1}{s^2+2s+2} \longleftrightarrow b(t) = e^{-t}\sin t\, u(t)$

$A_{11}(s) = \dfrac{s}{s^2+2s+2} \longrightarrow a_{11}(t) = e^{-t}(\cos t - \sin t)\,u(t)$

$A_{12}(s) = \dfrac{-2}{s^2+2s+2} \longrightarrow a_{12}(t) = -2e^{-t}\sin t\, u(t)$

$A_{21}(s) = \dfrac{1}{s^2+2s+2} \longrightarrow a_{21}(t) = e^{-t}\sin t\, u(t)$

$A_{22}(s) = \dfrac{s+2}{s^2+2s+2} \longrightarrow a_{22}(t) = e^{-t}(\cos t + \sin t)\,u(t)$

ENTÃO: $e^{Ft} = \begin{bmatrix} e^{-t}(\cos t - \sin t)\,u(t) & -2e^{-t}\sin t\, u(t) \\ e^{-t}\sin t\, u(t) & e^{-t}(\cos t+\sin t)\,u(t) \end{bmatrix} \longrightarrow \Phi = e^{FT} = \begin{bmatrix} e^{-T}(\cos T - \sin T) & -2e^{-T}\sin T \\ e^{-T}\sin T & e^{-T}(\cos T+\sin T) \end{bmatrix}$

$$\Gamma = \int_0^T e^{Ft}G\,dt = \int_0^T \begin{bmatrix} e^{-t}(\cos t - \sin t) \\ e^{-t}\sin t \end{bmatrix} dt = \begin{bmatrix} e^{-T}\sin T \\ \frac{1}{2}(1 - e^{-T}\cos T - e^{-T}\sin T) \end{bmatrix}$$

OBS.: $\int_0^T e^{-t}\cos t\,dt = \frac{1}{2}(1 - e^{-T}\cos T + e^{-T}\sin T)$

$\int_0^T e^{-t}\sin t\,dt = \frac{1}{2}(1 - e^{-T}\cos T - e^{-T}\sin T)$

$\Phi = \begin{bmatrix} a-b & -2b \\ b & a+b \end{bmatrix}$ $\Gamma = \begin{bmatrix} b \\ \frac{1}{2}(1-a-b) \end{bmatrix}$ $H = [1 \quad 1]$ $\begin{pmatrix} a = e^{-T}\cos T \\ b = e^{-T}\sin T \end{pmatrix}$

$D(z) = H(zI-\Phi)^{-1}\Gamma$

$$(zI-\Phi)^{-1} = \begin{bmatrix} z-a+b & 2b \\ -b & z-a-b \end{bmatrix}^{-1} = \frac{\begin{bmatrix} z-a-b & -2b \\ b & z-a+b \end{bmatrix}}{z^2-2az+a^2+b^2}$$

$$H(zI-\Phi)^{-1} = \frac{[1 \quad 1]\begin{bmatrix} z-a-b & -2b \\ b & z-a+b \end{bmatrix}}{z^2-2az+a^2+b^2} = \frac{[z-a \quad z-a-b]}{z^2-2az+a^2+b^2}$$

$$H(zI-\Phi)^{-1}\Gamma = \frac{[z-a \quad z-a-b]\begin{bmatrix} b \\ \frac{1}{2}(1-a-b) \end{bmatrix}}{z^2-2az+a^2+b^2} = \frac{\left(\frac{1}{2}-\frac{a}{2}+\frac{b}{2}\right)z + \left(\frac{a^2}{2}+\frac{b^2}{2}-\frac{a}{2}-\frac{b}{2}\right)}{z^2-2az+a^2+b^2}$$

$$bz - \cancel{ab} + \frac{z}{2} - \frac{a}{2} - \frac{b}{2} - \frac{az}{2} + \frac{a^2}{2} + \cancel{\frac{ab}{2}} - \frac{bz}{2} + \cancel{\frac{ab}{2}} + \frac{b^2}{2}$$

$$H(zI-\Phi)^{-1}\Gamma = \frac{1}{2}\cdot\frac{(1-a+b)z + (a^2+b^2-a-b)}{z^2-2az+a^2+b^2}$$

ENTÃO:
$$D(z) = \frac{0.5\left[\left(1+e^{-T}(\sin T - \cos T)\right)z + \left(e^{-2T} + e^{-T}(-\cos T - \sin T)\right)\right]}{z^2 - (2e^{-T}\cos T)z + e^{-2T}}$$

b) $D(z) = D(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}} = \dfrac{\frac{2}{T}\frac{(z-1)}{(z+1)} + 1}{\left(\frac{2}{T}\frac{(z-1)}{(z+1)}\right)^2 + 2\cdot\frac{2}{T}\frac{(z-1)}{(z+1)} + 2}$

$$D(z) = \frac{2(z-1)T(z+1) + T^2(z+1)^2}{4(z-1)^2 + 4(z-1)T(z+1) + 2T^2(z+1)^2}$$

$$D(z) = \frac{2Tz^2 - 2T + T^2z^2 + 2T^2z + T^2}{4z^2 - 8z + 4 + 4Tz^2 - 4T + 2T^2z^2 + 4T^2z + 2T^2}$$

$$D(z) = \frac{(T^2+2T)z^2 + 2T^2z + T^2 - 2T}{(2T^2+4T+4)z^2 + (4T^2-8)z + 2T^2 - 4T + 4}$$

→ SOLUÇÕES PROBLEMAS ④ (COMENTÁRIOS) E ⑤ — VER PÁGINAS SEGUINTES...

— " — " —

Lista #8 - Questao #4 - Comentario sobre as aproximacoes

```
V=[0.001 0.002 0.005 0.010 0.020 0.050 0.100 0.200 0.500 1.000];

M=[];

for k=1:length(V),

    T=V(k);

    nd=0.5*[0 1+exp(-T)*(sin(T)-cos(T)) exp(-2*T)+exp(-T)*(-cos(T)-sin(T))];

    dd=[1 -2*exp(-T)*cos(T) exp(-2*T)];

    nb=[T^2+2*T 2*T^2 T^2-2*T]/(2*T^2+4*T+4);

    db=[2*T^2+4*T+4 4*T^2-8 2*T^2-4*T+4]/(2*T^2+4*T+4);

    M=[M ; nd dd nb db [roots(nd)' roots(dd)' roots(nb)' roots(db)'] log([roots(nd)' roots(dd)'
roots(nb)' roots(db)']/T)];

end;

[V ; transpose(M)]

ans =

T

 0.0500             0.1000             0.2000             0.5000             1.0000

Discretizacao (numerador)

      0                  0                  0                  0                  0
 0.0488             0.0950             0.1801             0.3793             0.5554
-0.0464            -0.0860            -0.1474            -0.2276            -0.1865

Discretizacao (denominador)

 1.0000             1.0000             1.0000             1.0000             1.0000
-1.9001            -1.8006            -1.6048            -1.0646            -0.3975
 0.9048             0.8187             0.6703             0.3679             0.1353

Bilinear (numerador)

 0.0244             0.0475             0.0902             0.1923             0.3000
 0.0012             0.0045             0.0164             0.0769             0.2000
-0.0232            -0.0430            -0.0738            -0.1154            -0.1000

Bilinear (denominador)

 1.0000             1.0000             1.0000             1.0000             1.0000
-1.9001            -1.8009            -1.6066            -1.0769            -0.4000
 0.9049             0.8190             0.6721             0.3846             0.2000

Discretizacao (polos)

 0.9500 - 0.0475i   0.9003 - 0.0903i   0.8024 - 0.1627i   0.5323 - 0.2908i   0.1988 - 0.3096i
 0.9500 + 0.0475i   0.9003 + 0.0903i   0.8024 + 0.1627i   0.5323 + 0.2908i   0.1988 + 0.3096i

Bilinear (polos)

 0.9501 - 0.0476i   0.9005 - 0.0905i   0.8033 - 0.1639i   0.5385 - 0.3077i   0.2000 - 0.4000i
 0.9501 + 0.0476i   0.9005 + 0.0905i   0.8033 + 0.1639i   0.5385 + 0.3077i   0.2000 + 0.4000i

Discretizacao (zeros)

 0.9512             0.9048             0.8182             0.6001             0.3358

Bilinear (zeros)

-1.0000            -1.0000            -1.0000            -1.0000            -1.0000
 0.9512             0.9048             0.8182             0.6000             0.3333
```

Note que, `a medida em que T diminui, os resultados da aproximacao bilinear aproximam-se dos resultados da discretizacao no espaco de estados. A aproximacao bilinear possui um zero a mais, em z=-1.

Lista #8 - Questao #5

```
format long

V=[0.05 0.10 0.50 1.00 5.00 10.0];
M=[];

for k=1:length(V),

    T=V(k);

    % Mapeamento polos e zeros
    A=[roots([1 -(exp(-T)+exp(-4*T)) exp(-5*T)]); roots([1 -(exp(-2*T)+exp(-3*T)) exp(-5*T)])]

    % Forward Euler
    B=[roots([1 5*T-2 1-5*T+4*T^2]) ; roots([1 5*T-2 1-5*T+6*T^2])]

                          -(5*T+2)                          -(5*T+2)
    % Backward Euler
    C=[roots([T^2+5*T+4 -(2*T+5) 1]) ; roots([T^2+5*T+6 -(2*T+5) 1])]    <-- VER CORRECAO FEITA NA RESPOSTA (1C).
               4T^2+5*T+1                      6T^2+5T+1
    % Bilinear
    D=[roots([4*T^2+10*T+4 8*T^2-8 4*T^2-10*T+4]) ; roots([6*T^2+10*T+4 12*T^2-8 6*T^2-10*T+4])]

    % Equivalencia Degrau, ou Discretizacao (mesmo resultado)
    E=[roots([1 -1/3-5/3*exp(-3*T) 2/3*exp(-5*T)+exp(-3*T)-2/3*exp(-2*T)]) ; roots([1 -exp(-2*T)-
       exp(-3*T) exp(-5*T)])]

    M=[M [A ; B ; C ; D ; E]];
end;

[V ; M]
```

T					
0.05000000000000	0.10000000000000	0.50000000000000	1.00000000000000	5.00000000000000	10.00000000000000

Mapeamento Polos/Zeros

Z1	0.95122942450071	0.90483741803596	0.60653065971263	0.36787944117144	0.00673794699909	0.00004539992976
Z2	0.81873075307798	0.67032004603564	0.13533528323661	0.01831563888873	0.00000000206115	0.00000000000000
P1	0.90483741803596	0.81873075307798	0.36787944117144	0.13533528323661	0.00004539992976	0.00000000206115
P2	0.86070797642506	0.74081822068172	0.22313016014843	0.04978706836786	0.00000030590232	0.00000000000009

Forward Euler

Z1	0.95000000000000	0.90000000000000	-1.00000000000000	0	-19.00000000000000	-39.00000000000000
Z2	0.80000000000000	0.60000000000000	0.50000000000000	-3.00000000000000	-4.00000000000000	-9.00000000000000
P1	0.90000000000000	0.80000000000000	0	-2.00000000000000	-14.00000000000000	-29.00000000000000
P2	0.85000000000000	0.70000000000000	-0.50000000000000	-1.00000000000000	-9.00000000000000	-19.00000000000000

Backward Euler (INCORRETO)

Z1	~~0.95238095238095~~	~~0.90909090909091~~	~~0.66666666666667~~	~~0.50000000000000~~	~~0.16666666666667~~	~~0.09090909090909~~
Z2	~~0.24691358024691~~	~~0.24390243902439~~	~~0.22222222222222~~	~~0.20000000000000~~	~~0.11111111111111~~	~~0.07142857142857~~
P1	~~0.48780487804878~~	~~0.47619047619048~~	~~0.40000000000000~~	~~0.33333333333333~~	~~0.14285714285714~~	~~0.08333333333333~~
P2	~~0.32786885245902~~	~~0.32258064516129~~	~~0.28571428571429~~	~~0.25000000000000~~	~~0.12500000000000~~	~~0.07692307692308~~

Bilinear

Z1	0.95121951219512	0.90476190476190	0	0.33333333333333	-0.81818181818182	-0.90476190476190
Z2	0.81818181818182	0.66666666666667	0.60000000000000	-0.33333333333333	-0.42857142857143	-0.66666666666667
P1	0.90476190476190	0.81818181818182	0.33333333333333	0	-0.76470588235294	-0.87500000000000
P2	0.86046511627907	0.73913043478261	0.14285714285714	-0.20000000000000	-0.66666666666667	-0.81818181818182

Discretizacao

Z1	0.95201789030728	0.90782276370376	0.65548170606522	0.48971114916906	0.33342370093463	0.33333333745552
Z2	0.81582873706781	0.66020760409910	0.04973522751550	-0.07339936855596	-0.00008985776410	-0.00000000412203
P1	0.90483741803596	0.81873075307798	0.36787944117144	0.13533528323661	0.00004539992976	0.00000000206115
P2	0.86070797642506	0.74081822068172	0.22313016014843	0.04978706836786	0.00000030590232	0.00000000000009

OBS.: BACKWARD EULER (RESULTADO CORRIGIDO):

Z1	0.9524	0.9091	0.6667	0.5000	0.1667	0.0909
Z2	0.8333	0.7143	0.3333	0.2000	0.0476	0.0244
P1	0.9091	0.8333	0.5000	0.3333	0.0909	0.0476
P2	0.8696	0.7692	0.4000	0.2500	0.0625	0.0323

(051105)