

① a) $X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} = \frac{1}{1 - \frac{1}{3}z^{-1}}, |z| > \frac{1}{3} \quad \left(= \frac{z}{z - \frac{1}{3}} \right)$

b) $X(z) = \sum_{n=0}^{\infty} (-2)^n z^{-n} = \frac{1}{1 + 2z^{-1}} = \frac{z}{z + 2}, |z| > 2$

c) $\mathcal{Z}\{\delta(n)\} = 1 \in \mathcal{Z}\{x(n-k)\} = z^{-k} X(z)$

ENTÃO: $X(z) = 1 + 2z^{-1} + z^{-2}$

② a) $H(z) = \frac{A}{1 + \frac{1}{2}z^{-1}} + \frac{B}{1 - \frac{1}{3}z^{-1}}$

$A = \left(1 + \frac{1}{2}z^{-1}\right) H(z) \Big|_{z = -\frac{1}{2}} = \frac{1 + \frac{4}{3}z^{-1}}{1 - \frac{1}{3}z^{-1}} \Big|_{z = -\frac{1}{2}} = \frac{1 - \frac{8}{3}}{1 + \frac{2}{3}} = \frac{-\frac{5}{3}}{\frac{5}{3}} = -1$

$B = \left(1 - \frac{1}{3}z^{-1}\right) H(z) \Big|_{z = \frac{1}{3}} = \frac{1 + \frac{4}{3}z^{-1}}{1 + \frac{1}{2}z^{-1}} \Big|_{z = \frac{1}{3}} = \frac{1 + \frac{4}{3}}{1 + \frac{3}{2}} = \frac{\frac{7}{3}}{\frac{5}{2}} = \frac{14}{15}$

$H(z) = \frac{-1}{1 + \frac{1}{2}z^{-1}} + \frac{14}{1 - \frac{1}{3}z^{-1}}$

ROC: $|z| > \frac{1}{2}$ (SISTEMA CAUSAL)

$h(n) = -\left(-\frac{1}{2}\right)^n u(n) + 14\left(\frac{1}{3}\right)^n u(n)$

OPÇÃO:

$H(z) = \frac{(z + \frac{4}{3})z}{z^2 + \frac{1}{6}z - \frac{1}{6}}$

$\frac{H(z)}{z} = \frac{z + \frac{4}{3}}{z^2 + \frac{1}{6}z - \frac{1}{6}} = \frac{A}{z + \frac{1}{2}} + \frac{B}{z - \frac{1}{3}}$

$A = \frac{z + \frac{4}{3}}{z - \frac{1}{3}} \Big|_{z = -\frac{1}{2}} = \frac{-\frac{1}{2} + \frac{4}{3}}{-\frac{1}{2} - \frac{1}{3}} = -1$; $B = \frac{z + \frac{4}{3}}{z + \frac{1}{2}} \Big|_{z = \frac{1}{3}} = \frac{\frac{5}{3}}{\frac{2+3}{6}} = 2$

$H(z) = -1 \cdot \frac{z}{z + \frac{1}{2}} + 2 \cdot \frac{z}{z - \frac{1}{3}}$

ROC: $|z| > \frac{1}{2}$ (SISTEMA CAUSAL), $h(n) = -\left(-\frac{1}{2}\right)^n u(n) + 2\left(\frac{1}{3}\right)^n u(n)$

b) $H(z) = \frac{(z + \frac{4}{3})z}{(z^2 + \frac{1}{6}z - \frac{1}{6})} \cdot \frac{z}{z-1} = \frac{(z + \frac{4}{3})z^2}{(z + \frac{1}{2})(z - \frac{1}{3})(z-1)}$

$\frac{H(z)}{z} = \frac{(z + \frac{4}{3})z}{(z + \frac{1}{2})(z - \frac{1}{3})(z-1)} = \frac{-\frac{1}{3}}{z + \frac{1}{2}} + \frac{(-1)}{z - \frac{1}{3}} + \frac{\cancel{\frac{7}{3}}}{z-1}$

$A = \frac{(z + \frac{4}{3})z}{(z - \frac{1}{3})(z-1)} \Big|_{z = -\frac{1}{2}} = \frac{\left(\frac{-3+8}{6}\right)\left(-\frac{1}{2}\right)}{\left(\frac{-3-2}{6}\right)\left(-\frac{3}{2}\right)} = (-1)\left(\frac{1}{3}\right) = -\frac{1}{3}$

$B = \frac{(z + \frac{4}{3})z}{(z + \frac{1}{2})(z-1)} \Big|_{z = \frac{1}{3}} = \frac{\frac{5}{3} \cdot \frac{1}{3}}{\frac{3+2}{6} \cdot \left(-\frac{2}{3}\right)} = -1$

$y(n) = \left(-\frac{1}{3}\left(-\frac{1}{2}\right)^n - \left(\frac{1}{3}\right)^n + \frac{7}{3}\right) u(n)$

$C = \frac{(z + \frac{4}{3})z}{(z + \frac{1}{2})(z - \frac{1}{3})} \Big|_{z = +1} = \frac{\frac{7}{3} \cdot 1}{\frac{3}{2} \cdot \left(\frac{2}{3}\right)} = \frac{7}{3}$

c) $1 + \frac{4}{3}z^{-1}$
 $-1 - \frac{1}{6}z^{-1} + \frac{1}{6}z^{-2}$

 $\frac{7}{6}z^{-1} + \frac{1}{6}z^{-2}$
 $-\frac{7}{6}z^{-1} - \frac{7}{36}z^{-2} + \frac{7}{36}z^{-3}$

 $-\frac{1}{36}z^{-2} + \frac{7}{36}z^{-3}$
 $\frac{1}{36}z^{-2} + \frac{1}{216}z^{-3} - \frac{1}{216}z^{-4}$

 $\frac{43}{216}z^{-3} - \frac{1}{216}z^{-4}$

$1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}$
 $1 + \frac{7}{6}z^{-1} - \frac{1}{36}z^{-2} + \frac{43}{216}z^{-3} + \dots$

n	h(n)
0	1
1	7/6
2	-1/36
3	43/216

ITEM (a)

ITEM (b):

$1 + \frac{4}{3}z^{-1}$
 $-1 + \frac{5}{6}z^{-1} + \frac{1}{3}z^{-2} - \frac{1}{6}z^{-3}$

 $\frac{13}{6}z^{-1} + \frac{1}{3}z^{-2} - \frac{1}{6}z^{-3}$
 $-\frac{13}{6}z^{-1} + \frac{65}{36}z^{-2} + \frac{13}{18}z^{-3} - \frac{13}{36}z^{-4}$

 $\frac{77}{36}z^{-2} + \frac{10}{18}z^{-3} - \frac{13}{36}z^{-4}$
 $-\frac{77}{36}z^{-2} + \frac{385}{216}z^{-3} + \frac{77}{108}z^{-4} - \frac{77}{216}z^{-5}$

 $\frac{505}{216}z^{-3} + \dots$

n	y(n)
0	1
1	13/6
2	77/36
3	505/216

3

$y(k) - 3y(k-1) + 2y(k-2) = 2u(k-1) - 2u(k-2)$

$y(-1) = y(-2) = 0$ (CONDIÇÕES INICIAIS NULAS). ENTÃO:

$Y(z)(1 - 3z^{-1} + 2z^{-2}) = U(z)(2z^{-1} - 2z^{-2})$

$Y(z) = \frac{2(z-1)}{z^2 - 3z + 2} \cdot U(z) = \frac{2(z-1)}{(z-1)(z-2)} U(z) = \frac{2}{z-2} U(z)$

CÁLCULO DE $U(z)$: $\frac{d}{dz} \left(\frac{z}{z-1} = \sum_{k=0}^{\infty} z^{-k} \right)$

$\frac{d}{dz} \left(\frac{z}{z-1} \right) = - \sum_{k=0}^{\infty} k z^{-k-1}$

$-z \cdot \frac{d}{dz} \left(\frac{z}{z-1} \right) = \sum_{k=0}^{\infty} k z^{-k} = U(z)$

ENTÃO: $U(z) = -z \cdot \left[\frac{(z-1) - z}{(z-1)^2} \right] = \frac{z}{(z-1)^2}$

PORTANTO:

$Y(z) = \frac{2z}{(z-2)(z-1)^2}$

$\frac{Y(z)}{z} = \frac{A}{(z-2)} + \frac{B}{(z-1)^2} + \frac{C}{(z-1)} = \frac{2}{z-2} - \frac{2}{(z-1)^2} - \frac{2}{z-1}$

$A = \frac{2}{(z-1)^2} \Big|_{z=2} = 2$

$B = \frac{2}{(z-2)} \Big|_{z=1} = -2$

$C = \frac{d}{dz} \left(\frac{2}{z-2} \right) \Big|_{z=1} = \frac{-2}{(z-2)^2} \Big|_{z=1} = -2$

ENTÃO: $y(k) = (2(2)^k - 2k - 2) u(k) = 2(2^k - k - 1) u(k)$