UFRGS / DEL EEL760 — CONTROLE LINEAR II — SEGUNDO SEMESTRE DE 2005

LISTA DE EXERCÍCIOS #6 — GABARITO

EXERCÍCIO #1

$x_1 = \theta$ E $x_2 = \theta'$

$f_1(x_1, x_2, u) = x_1' = x_2$ E $f_2(x_1, x_2, u) = x_2' = \frac{meg}{J} \operatorname{sen} x_1 + \frac{u}{J}$

PONTO DE EQUILÍBRIO: (PARA $u_0 = 0$): $f_1(x_{10}, x_{20}, u_0) = 0 \longrightarrow x_{20} = 0$

$f_2(x_{10}, x_{20}, u_0) = 0 \longrightarrow \operatorname{sen} x_{10} = 0$

$x_{10} = 0$ (ESCOLHA MAIS SIMPLES)

$\frac{\partial f_1}{\partial x_1} = 0$; $\frac{\partial f_1}{\partial x_2} = 1$; $\frac{\partial f_1}{\partial u} = 0$

$\frac{\partial f_2}{\partial x_1} = \frac{meg}{J} \cos x_1 \longrightarrow \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1 = x_{10}} = \frac{meg}{J}$; $\frac{\partial f_2}{\partial x_2} = 0$; $\frac{\partial f_2}{\partial u} = \frac{1}{J}$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{meg}{J} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} u$$

$$y = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

EXERCÍCIO #2

$$\begin{bmatrix} x_i' \\ x' \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}}_{F_i} \begin{bmatrix} x_i \\ x \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{G_i} u - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$

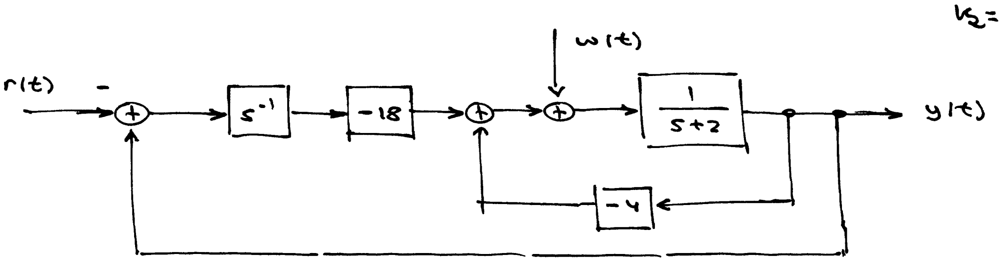
a) $F_i - G_i K = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [K_1 \quad K_2] = \begin{bmatrix} 0 & 1 \\ -K_1 & -2-K_2 \end{bmatrix}$

$|sI - F_i + G_i K| = \begin{vmatrix} s & -1 \\ K_1 & s+2+K_2 \end{vmatrix} = s^2 + (K_2 + 2)s + K_1$

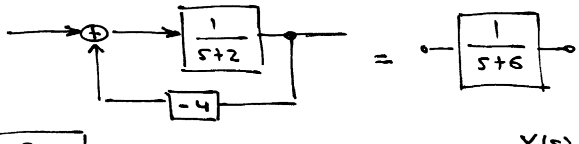
$\alpha_c(s) = (s+3+3j)(s+3-3j) = s^2 + 6s + 18 \longrightarrow K_1 = 18 = K_i$

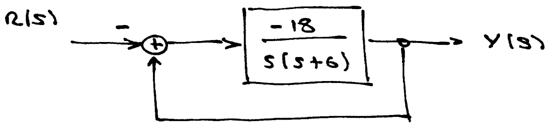
$K_2 = 4 = K_0$

b)



c) $\frac{Y(s)}{R(s)}$: $(W(s) = 0)$

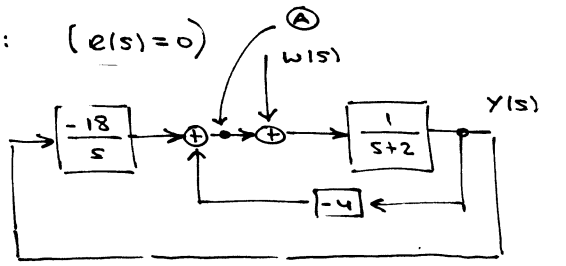




, ENTÃO: $\frac{Y(s)}{-R(s)} = \frac{\frac{-18}{s(s+6)}}{1 + \frac{18}{s(s+6)}} = \frac{-18}{s^2+6s+18}$

$\frac{Y(s)}{R(s)} = \frac{18}{s^2+6s+18}$

$\frac{Y(s)}{W(s)}$: $(R(s) = 0)$



NO PONTO A: TEMOS $(-18s^{-1} - 4)Y(s)$

ENTÃO: $\frac{(-18s^{-1} - 4)Y(s) + W(s)}{(s+2)} = Y(s)$

$W(s) = Y(s)(s + 2 + 18s^{-1} + 4) = Y(s)\left(s + 6 + \frac{18}{s}\right)$

$\frac{Y(s)}{W(s)} = \frac{s}{s^2+6s+18}$

exercício # 3

POR EXEMPLO , $F = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\quad H = [1 \ \ 0]$

$\mathcal{C} = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$ $\quad \det \mathcal{C} = 0$

$\Theta = \begin{bmatrix} 1 & 0 \\ -5 & -3 \end{bmatrix}$ $\quad \det \Theta = -3$

FORMA CANÔNICA OBSERVÁVEL: $t_2 = \Theta^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -5/3 & -1/3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1/3 \end{bmatrix}$

$t_1 = F t_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$T = \begin{bmatrix} 1 & 0 \\ 0 & -1/3 \end{bmatrix}$ $\quad T^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$

$F_{co} = T^{-1} F T = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1/3 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ -6 & 0 \end{bmatrix}$

$G_{co} = T^{-1} G = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$H_{co} = HT = [1 \ \ 0]\begin{bmatrix} 1 & 0 \\ 0 & -1/3 \end{bmatrix} = [1 \ \ 0]$

A PARTIR DE F, G, H:

$$G(s) = H(sI - F)^{-1} G = \frac{[1 \ \ 0]\begin{bmatrix} s & -3 \\ 2 & s+5 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}{s^2 + 5s + 6} = \frac{[s \ \ -3]\begin{bmatrix} 1 \\ -1 \end{bmatrix}}{s^2 + 5s + 6}$$

$G(s) = \dfrac{s+3}{s^2 + 5s + 6}$ $\longrightarrow$ A NÃO-CONTROLABILIDADE DO SISTEMA REPRESENTADO POR F, G E H ESTÁ RELACIONADA AO CANCELAMENTO ENTRE O PÓLO E O ZERO EM $s = -3$.

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