CONTROLE LINEAR II - SEGUNDO SEMESTRE DE 2005 - GABARITO DA LISTA DE ENERCICIOS #5

EXERCICIO #1

a) $F - GK = \begin{bmatrix} -2-K_1 & -2-K_2 \\ 1 & 0 \end{bmatrix}$

$\alpha_c(s) = (s+2+j)(s+2-j) = s^2 + 4s + 5$

$K_1 + 2 = 4$

$K_2 + 2 = 5$

$K = \begin{bmatrix} 2 & 3 \end{bmatrix}$

b) $F - LH = \begin{bmatrix} -2-\ell_1 & -2-4\ell_1 \\ 1-\ell_2 & -4\ell_2 \end{bmatrix}$

$\alpha_e(s) = (s+5+j)(s+5-j) = s^2 + 10s + 26$

$\det(sI - F + LH) = \begin{bmatrix} s+2+\ell_1 & 2+4\ell_1 \\ \ell_2 - 1 & s+4\ell_2 \end{bmatrix} = s^2 + (\ell_1 + 4\ell_2 + 2)s + 4\ell_1 + 6\ell_2 + 2$

$\begin{cases} \ell_1 + 4\ell_2 = 8 \\ 2\ell_1 + 3\ell_2 = 12 \end{cases}$

$\ell_2 = \frac{4}{5} \qquad \ell_1 = \frac{24}{5} \qquad L = \begin{bmatrix} 4.8 \\ 0.8 \end{bmatrix}$

c) $\hat{x}' = (F - GK - LH)\hat{x} + Ly$

$u = -K\hat{x}$

ONDE : $F - GK - LH = \begin{bmatrix} -8.8 & -24.2 \\ 0.2 & -3.2 \end{bmatrix} \qquad L = \begin{bmatrix} 4.8 \\ 0.8 \end{bmatrix}$

$-K = \begin{bmatrix} -2 & -3 \end{bmatrix}$

d) $D(s) = -K(sI - F + GK + LH)^{-1}L = \dfrac{-\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} s+3.2 & -24.2 \\ 0.2 & s+8.8 \end{bmatrix}}{s^2 + 12s + 33} \begin{bmatrix} 4.8 \\ 0.8 \end{bmatrix}$

$= \dfrac{-\begin{bmatrix} 2s+7 & 3s-22 \end{bmatrix} \begin{bmatrix} 4.8 \\ 0.8 \end{bmatrix}}{s^2 + 12s + 33}$

$D(s) = \dfrac{-12\left(s + \frac{4}{3}\right)}{s^2 + 12s + 33}$

e) $G(s) = H(sI - F)^{-1}G = \begin{bmatrix} 1 & 4 \end{bmatrix} \dfrac{\begin{bmatrix} s & -2 \\ 1 & s+2 \end{bmatrix}}{s^2 + 2s + 2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \dfrac{\begin{bmatrix} 1 & 4 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2 + 2s + 2} = \dfrac{s+4}{s^2 + 2s + 2}$

f) $$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 - G(s)D(s)}$$

$$G(s) = \frac{s+4}{s^2+2s+2} = \frac{n_G(s)}{d_G(s)}$$

$$D(s) = \frac{-12\left(s+\frac{4}{3}\right)}{s^2+12s+33} = \frac{n_D(s)}{d_D(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{n_G(s)}{d_G(s)}}{1 - \frac{n_G(s)}{d_G(s)}\frac{n_D(s)}{d_D(s)}} = \frac{n_G(s)\,d_D(s)}{d_G(s)\,d_D(s) - n_G(s)\,n_D(s)}$$

$$= \frac{(s+4)(s^2+12s+33)}{(s^2+2s+2)(s^2+12s+33) - (s+4)(-12)\left(s+\frac{4}{3}\right)}$$

$$\frac{Y(s)}{R(s)} = \frac{s^3+16s^2+81s+132}{s^4+14s^3+71s^2+154s+130} \longrightarrow \text{roots}([1\ \ 14\ \ 71\ \ 154\ \ 130]):$$

$$-5+j\ ;\ -5-j\ ;\ -2+j\ ;\ -2-j$$

g) $$\lim_{s\to 0} \frac{Y(s)}{R(s)} = \frac{132}{130} \longrightarrow \bar{N} = \frac{130}{132} = 0.9848$$

ALTERNATIVO: $H = 0 \longrightarrow \alpha = H(F-GK)^{-1}G\left(1 - K(F-LH)^{-1}G\right) = -1.0154$

$$\bar{N} = \frac{-1}{\alpha} \longrightarrow \bar{N} = 0.9848$$

EXERCICIO #2

a) K É O MESMO DO PROBLEMA #1

b) $z = Px$, $P = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$; $P^{-1} = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = T$

$$F_z = T^{-1}FT = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = \left[\begin{array}{c|c} \overset{f_{aa}}{2} & \overset{f_{ab}}{-10} \\ \hline \underset{f_{ba}}{1} & \underset{f_{bb}}{-4} \end{array}\right]$$

$$\left.\begin{array}{l} \det(sI - F_{bb} + LF_{ab}) = s + 4 - 10L \\ \alpha_e(s) = s + 6 \end{array}\right\} \quad L = -0.2$$

DEPENDE DA ESCOLHA DA MATRIZ P.

c) $x_c' = F_r x_c + G_r y$

$u = H_r x_c + J_r y$

PARA CALCULAR AS MATRIZES F_r, G_r, H_r E J_r, LEMBRE QUE

$$u = -[K_1\ \ K_2]\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = -[K_1\ \ K_2]\,T\begin{bmatrix} z_1 \\ \hat{z}_2 \end{bmatrix} = -[\overset{K_{za}}{2}\ \ |\ \ \overset{K_{zb}}{-5}]\begin{bmatrix} z_1 \\ \hat{z}_2 \end{bmatrix}$$

$$z' = \underbrace{T^{-1}FT}_{F_z}z + \underbrace{T^{-1}G}_{G_z}u$$

$$y = \underbrace{HT}_{H_z}z$$

$$G_z = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \hline 0 \end{bmatrix} \begin{matrix} \leftarrow G_a \\ \leftarrow G_b \end{matrix} \quad ; \quad H_z = [1 \quad 4]\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = [1 \quad 0]$$

$$F_r = F_{bb} - LF_{ab} - (\underbrace{G_b}_{0} - LG_a)K_{zb} = -4 - 2 + 1 = -5$$

$$G_r = \underbrace{F_rL}_{1} + \underbrace{F_{ba}}_{1} - \underbrace{LF_{aa}}_{+0.4} - \underbrace{(\underbrace{G_b}_{0} - LG_a)K_{za}}_{-0.4} = 2$$

$$H_r = -K_{zb} = 5$$

$$J_r = -K_{za} - K_{zb}L = -2 - 1 = -3$$

$$x_c' = -5x_c + 2y$$

$$u = 5x_c - 3y$$

d) $D(s) = H_r(sI - F_r)^{-1}G_r + J_r = \dfrac{10}{s+5} - 3 = \dfrac{-3s-5}{s+5}$

e) G(s) É O MESMO DO PROBLEMA #1

f) $\dfrac{Y(s)}{R(s)} = \dfrac{n_G(s)\,d_D(s)}{d_G(s)\,d_D(s) - n_G(s)\,n_D(s)} = \dfrac{(s+4)(s+5)}{(s^2+2s+2)(s+5) - (s+4)(-1)(3s+5)}$

$\dfrac{Y(s)}{R(s)} = \dfrac{s^2+9s+20}{s^3+10s^2+29s+30} \longrightarrow$ roots([1 10 29 30]) :

$-6 \; ; \; -2+j \; ; \; -2-j$

g) $\lim_{s \to 0} \dfrac{Y(s)}{R(s)} = \dfrac{20}{30} \longrightarrow \bar{N} = 1.5$

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