a) $$\frac{Y(s)}{U(s)} = \frac{10}{s(s+1)}$$

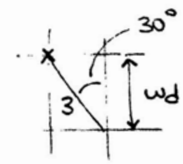
$$y'' + y' = 10u$$

$$y' = x_2$$

$$x_2' = -x_2 + 10u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

ENTÃO: $$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

b) $\omega_n = 3$; $\xi = 0.5 = \sin 30^\circ \longrightarrow$



$$\omega_d = \omega_n \sqrt{1-\xi^2} = \frac{\sqrt{3}}{2}\omega_n = \frac{3\sqrt{3}}{2}$$

$$-\sigma = \frac{-3}{2}$$

$$\alpha_c(s) = \left(s + \frac{3}{2} + \frac{3\sqrt{3}}{2}j\right)\left(s + \frac{3}{2} - \frac{3\sqrt{3}}{2}j\right) = s^2 + 3s + 9$$

$$F - GK = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 10 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -10k_1 & -1-10k_2 \end{bmatrix}$$

$$\det\left(sI - F + GK\right) = s^2 + (1+10k_2)s + 10k_1$$

$$\boxed{\begin{matrix} k_2 = 0.2 \\ k_1 = 0.9 \end{matrix}}$$

$$\begin{array}{r} 225\,+ \\ \times 4 \\ \hline 900 \end{array}$$

c) $\omega_n = 15$; $\xi = 0.5 \longrightarrow \alpha_e(s) = \left(s + \frac{15}{2} + \frac{15\sqrt{3}}{2}j\right)\left(s + \frac{15}{2} - \frac{15\sqrt{3}}{2}j\right)$

$$\alpha_e(s) = s^2 + 15s + 225$$

$$F - LH = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} -\ell_1 & 1 \\ -\ell_2 & -1 \end{bmatrix}$$

$$\det(sI - F + LH) = (s+\ell_1)(s+1) + \ell_2 = s^2 + (\ell_1 + 1)s + \ell_1 + \ell_2$$

$$\boxed{\begin{matrix} \ell_1 = 14 \\ \ell_2 = 211 \end{matrix}}$$

d) $$D(s) = -K\left(sI - F + GK + LH\right)^{-1} L$$

$$F - GK - LH = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 10 \end{bmatrix} \begin{bmatrix} 0.9 & 0.2 \end{bmatrix} - \begin{bmatrix} 14 \\ 211 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} -14 & 1 \\ -220 & -3 \end{bmatrix}$$

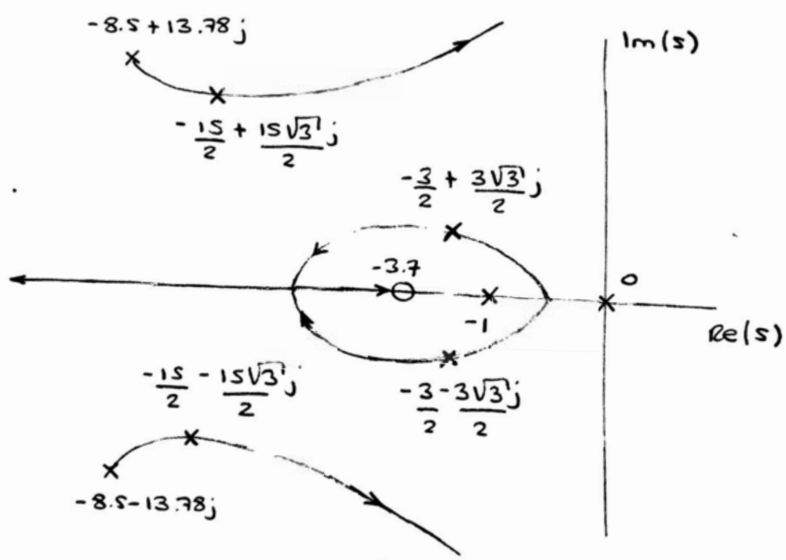
$$D(s) = \frac{-\begin{bmatrix} 0.9 & 0.2 \end{bmatrix} \begin{bmatrix} s+3 & 1 \\ -220 & s+14 \end{bmatrix} \begin{bmatrix} 14 \\ 211 \end{bmatrix}}{s^2 + 17s + 272}$$

$$\boxed{D(s) = \frac{-(54.8s + 202.5)}{s^2 + 17s + 272}}$$

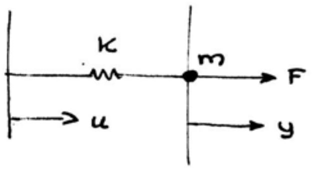
e) RAÍZES: $-8.5 \pm 13.78j$ (PÓLOS) } COMPENSADOR
-3.70 (ZERO)

0 E -1 (PÓLOS) — PLANTA

ROOT - LOCUS:



EXERCICIO #2



ACELERAÇÃO: $y'' = \frac{F}{m}$; $F = k(u-y)$

$$y'' = -\frac{ky}{m} + \frac{ku}{m}$$

a) $y'' + \frac{ky}{m} = \frac{ku}{m}$

$$\frac{Y(s)}{U(s)} = \frac{\frac{k}{m}}{s^2 + \frac{k}{m}}$$

FORMA CANÔNICA CONTROLÁVEL:
$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & -\frac{k}{m} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & \frac{k}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\left(\frac{k}{m} = 900 \text{ rad/sec} \right)$$

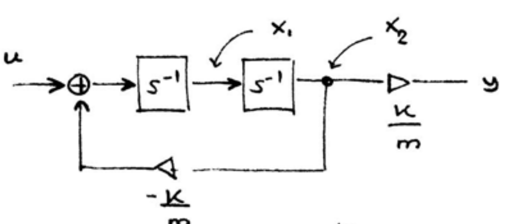
b) $\alpha_e(s) = (s + 100 + 100j)(s + 100 - 100j) = s^2 + 200s + 20000$

$$(F - LH) = \begin{bmatrix} 0 & -900 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} [0 \quad 900] = \begin{bmatrix} 0 & -900(\ell_1 + 1) \\ 1 & -900\ell_2 \end{bmatrix}$$

$$|sI - F + LH| = \begin{vmatrix} s & 900(\ell_1 + 1) \\ -1 & s + 900\ell_2 \end{vmatrix} = s^2 + 900\ell_2 s + 900(\ell_1 + 1)$$

$\alpha_e(s) = |sI - F + LH| \Rightarrow \ell_1 = \frac{191}{9}$; $\ell_2 = \frac{2}{9}$; $\boxed{L = \frac{1}{9}\begin{bmatrix} 191 \\ 2 \end{bmatrix}}$

c)



$$y' = \frac{k}{m} x_1 = \begin{bmatrix} \frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

O SISTEMA $x' = \begin{bmatrix} 0 & -900 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$ É OBSERVÁVEL?

$$y_2 = [900 \quad 0] x$$

$\mathcal{O} = \begin{bmatrix} 900 & 0 \\ 0 & -900^2 \end{bmatrix} \longrightarrow \det \mathcal{O} = -900^3 \neq 0 \longrightarrow$ O SISTEMA É OBSERVÁVEL. PORTANTO, x_1 E x_2 PODEM SER ESTIMADAS A PARTIR DE y'.

d) $\alpha_c(s) = (s+20+20j)(s+20-20j) = s^2 + 40s + 800$

$$F - GK = \begin{bmatrix} 0 & -900 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} -k_1 & -900-k_2 \\ 1 & 0 \end{bmatrix}$$

$k_1 = 40$; $k_2 = -100$ $\longrightarrow$ $\boxed{K = [40 \quad -100]}$

e) $\alpha_c(s) = (s+200+200j)(s+200-200j) = s^2 + 400s + 80000$

$k_1 = 400$

$k_2 = 79100$

$K = [400 \quad 79100]$ $\longrightarrow$ NÃO SERIA RAZOÁVEL. OS GANHOS SÃO MUITO ALTOS (EM RELAÇÃO AO ITEM (d)). ALÉM DISSO, OS PÓLOS DO ERRO DE ESTIMAÇÃO SÃO $-100 \pm 100j$, E O CONTROLADOR DEVERIA SER MAIS LENTO.

f) COMPENSADOR:

$K = [40 \quad -100]$

$$L = \begin{bmatrix} \frac{191}{9} \\ \frac{2}{9} \end{bmatrix}$$

$$F - GK - LH = \begin{bmatrix} 0 & -900 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 40 & -100 \end{bmatrix} - \begin{bmatrix} \frac{191}{9} \\ \frac{2}{9} \end{bmatrix} [0 \quad 900]$$

$$= \begin{bmatrix} -40 & -19900 \\ 1 & -200 \end{bmatrix}$$

$$(sI - F + GK + LH)^{-1} = \begin{bmatrix} s+40 & 19900 \\ -1 & s+200 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} s+200 & -19900 \\ 1 & s+40 \end{bmatrix}}{s^2 + 240s + 27900}$$

FUNÇÃO DE TRANSFERÊNCIA:

$$\frac{U(s)}{Y(s)} = -K(sI - F + GK + LH)^{-1}L = -[40 \quad -100] \frac{\begin{bmatrix} s+200 & -19900 \\ 1 & s+40 \end{bmatrix}}{s^2 + 240s + 27900} \begin{bmatrix} \frac{191}{9} \\ \frac{2}{9} \end{bmatrix}$$

$$\boxed{\frac{U(s)}{Y(s)} = \frac{-826.67\,(s - 12.24462)}{s^2 + 240s + 27900}}$$

EQUAÇÕES DE ESTADO DO COMPENSADOR:

$$\hat{x}' = \begin{bmatrix} -40 & -19900 \\ 1 & -200 \end{bmatrix} \hat{x} + \begin{bmatrix} \frac{191}{9} \\ \frac{2}{9} \end{bmatrix} y$$

$$u = -[40 \quad -100]\,\hat{x}$$

$$x_1' + \sigma x_1 = u \qquad\qquad \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -\sigma & 0 \\ \sigma & -\sigma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u.$$
$$x_2' + \sigma x_2 = x_1 \longrightarrow$$

a) $F - GK = \begin{bmatrix} -\sigma & 0 \\ \sigma & -\sigma \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [k_1 \;\; k_2] = \begin{bmatrix} -\sigma - k_1 & -k_2 \\ \sigma & -\sigma \end{bmatrix}$ $\qquad (u = -k_1 x_1 - k_2 x_2)$

$$|sI - F + GK| = \begin{vmatrix} s + \sigma + k_1 & k_2 \\ -\sigma & s + \sigma \end{vmatrix} = s^2 + (2\sigma + k_1)s + \sigma k_2 + \sigma^2 + \sigma k_1$$

$$\alpha_c(s) = (s + 2\sigma + 2\sigma j)(s + 2\sigma - 2\sigma j) = s^2 + 4\sigma + 8\sigma^2$$

$k_1 = 2\sigma$
$k_2 = 5\sigma$ $\longrightarrow$ $\boxed{K = [2\sigma \;\; 5\sigma]}$

b) $y = [0 \;\; 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$F - LH = \begin{bmatrix} -\sigma & 0 \\ \sigma & -\sigma \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} [0 \;\; 1] = \begin{bmatrix} -\sigma & -\ell_1 \\ \sigma & -\sigma - \ell_2 \end{bmatrix}$$

$$|sI - F + LH| = \begin{vmatrix} s + \sigma & \ell_1 \\ -\sigma & s + \sigma + \ell_2 \end{vmatrix} = s^2 + (2\sigma + \ell_2)s + \sigma^2 + \sigma\ell_2 + \sigma\ell_1$$

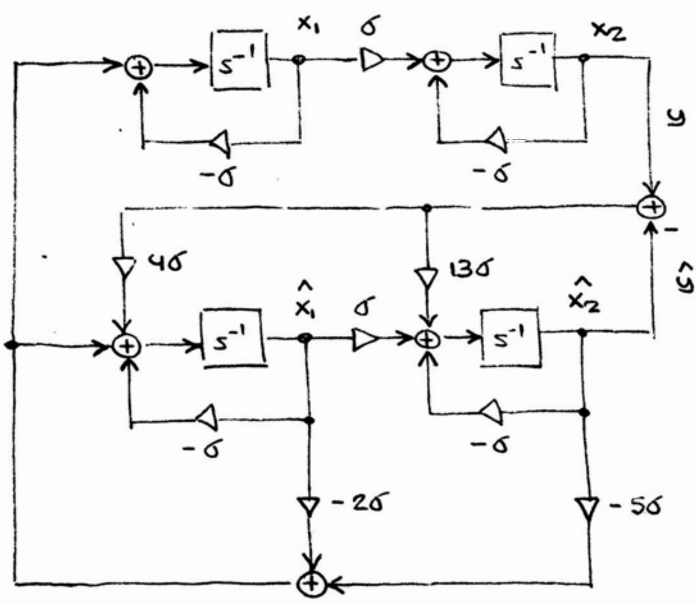
$$\alpha_e(s) = (s + 3\sigma + 3\sigma j)(s + 3\sigma - 3\sigma j) = s^2 + 6\sigma s + 18\sigma^2$$

$\ell_2 = 4\sigma$
$\ell_1 = 13\sigma$ $\longrightarrow$ $\boxed{L = \begin{bmatrix} 4\sigma \\ 13\sigma \end{bmatrix}}$

c). $x' = Fx + Gu$
$y = Hx$

$\hat{x}' = F\hat{x} + Gu + L(y - H\hat{x})$
$u = -K\hat{x}$

IMPLEMENTAÇÃO :



EXERCICIO #4

a) $$x' = \begin{bmatrix} -0.0895 & -0.286 & 0 \\ -0.0439 & -0.272 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0.0145 \\ -0.0122 \\ 0 \end{bmatrix} u$$

$$y = [\,0 \quad 0 \quad 1\,]\, x$$

$$G(s) = \frac{Y(s)}{U(s)} = H(sI-F)^{-1}G$$

$$\left[\begin{array}{cc|c} s+0.0895 & 0.286 & 0 \\ 0.0439 & s+0.272 & 0 \\ \hline 0 & -1 & s \end{array}\right]^{-1} = ?$$

$$\begin{bmatrix} A & 0 \\ C & B \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} & 0 \\ -B^{-1}CA^{-1} & B^{-1} \end{bmatrix}$$

$$A^{-1} = \frac{\begin{bmatrix} s+0.272 & -0.286 \\ -0.0439 & s+0.0895 \end{bmatrix}}{s^2+0.3615s+0.01179} \qquad B^{-1} = \frac{1}{s}$$

$$-B^{-1}CA^{-1} = -\frac{1}{s}[\,0 \quad -1\,] \frac{\begin{bmatrix} s+0.272 & -0.286 \\ -0.0439 & s+0.0895 \end{bmatrix}}{s^2+0.3615s+0.01179} = \frac{[\,-0.0439 \quad s+0.0895\,]}{(s^2+0.3615s+0.01179)s}$$

$$H(sI-F)^{-1} = [\,0 \quad 0 \quad 1\,] \left[\begin{array}{cc|c} \multicolumn{2}{c|}{A^{-1}} & 0 \\ \hline \frac{-0.0439}{(s^2+0.3615s+0.01179)s} & \frac{s+0.0895}{(s^2+0.3615s+0.01179)s} & \frac{1}{s} \end{array}\right]$$

$$= \begin{bmatrix} \frac{-0.0439}{(s^2+0.3615s+0.01179)s} & \frac{s+0.0895}{(s^2+0.3615s+0.01179)s} & \frac{1}{s} \end{bmatrix}$$

$$H(sI-F)^{-1}G = \frac{-0.0145 \times 0.0439 - 0.0122s - 0.0895 \times 0.0122}{s(s^2+0.3615s+0.01179)}$$

$$\boxed{G(s) = \frac{-0.0122(s+0.14168)}{s^3+0.3615s^2+0.01179s}} = \frac{-0.0122(s+0.14168)}{s(s+0.32526)(s+0.03624)}$$

PÓLOS EM MALHA ABERTA : $s_1 = 0$

$s_2 = -0.32526$

$s_3 = -0.03624$

b) $u = -Kx$

$$\alpha_c(s) = (s+0.2)\underbrace{(s+0.2+0.2j)(s+0.2-0.2j)}_{s^2+0.4s+0.08} = s^3+0.6s^2+0.16s+0.016$$

$$\alpha_c(F) = \begin{bmatrix} 0.00764 & -0.01773 & 0 \\ -0.00272 & -0.00367 & 0 \\ -0.01047 & 0.08334 & 0.016 \end{bmatrix}$$

$$\mathcal{C} = [\,G \quad FG \quad F^2G\,] = \begin{bmatrix} 0.0145 & 0.00219 & -0.00096 \\ -0.0122 & 0.00268 & -0.00083 \\ 0 & -0.0122 & 0.00268 \end{bmatrix}$$

$$K = [0 \;\; 0 \;\; 1]\,\mathcal{C}^{-1}\alpha_c(F) \longrightarrow \boxed{K = [\,0.27665 \;\; -19.22038 \;\; -9.25685\,]}$$

c) $$\alpha_e(s) = (s+0.4)\underbrace{(s+0.4+0.4j)(s+0.4-0.4j)}_{s^2+0.8s+0.32} = s^3+1.2s^2+0.64s+0.128$$

$$\alpha_e(F) = \begin{bmatrix} 0.08902 & -0.09298 & 0 \\ -0.01427 & 0.02969 & 0 \\ -0.03681 & 0.40014 & 0.128 \end{bmatrix}$$

$$\mathcal{O} = \begin{bmatrix} H \\ HF \\ HF^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -0.0439 & -0.272 & 0 \end{bmatrix}$$

$$L = \alpha_e(F)\,\mathcal{O}^{-1}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \longrightarrow \boxed{L = \begin{bmatrix} -2.02778 \\ 0.32509 \\ 0.83850 \end{bmatrix}}$$

d) $$F - GK - LH = \begin{bmatrix} -0.09351 & -0.00730 & 2.16200 \\ -0.04052 & -0.50649 & -0.43803 \\ 0 & 1 & -0.83850 \end{bmatrix}$$

$$\boxed{\begin{aligned} \hat{x}' &= (F - GK - LH)\hat{x} + Ly \\ u &= -K\hat{x} \end{aligned}}$$

$$\frac{U(s)}{Y(s)} = D_c(s) = -K(sI - F + GK + LH)^{-1}L$$

$$\boxed{D_c(s) = \frac{14.57127\,(s^2 + 0.56713s + 0.08132)}{s^3 + 1.43850s^2 + 0.98819s + 0.16804}}$$

← O CÁLCULO MANUAL É SEMELHANTE AO DO PROBLEMA #5 DA LISTA #3 E FOI OMITIDO. ESTE RESULTADO FOI OBTIDO NO MATLAB ATRAVÉS DE

```
[numD, denD] = ss2tf(F-GK-LH, L, -K, 0);
```

a) $F - LH = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} [1 \quad 1] = \begin{bmatrix} -1-\ell_1 & -\ell_1 \\ -\ell_2 & -4+\ell_2 \end{bmatrix}$

$$\underbrace{\begin{vmatrix} s+1+\ell_1 & \ell_1 \\ \ell_2 & s+4+\ell_2 \end{vmatrix}}_{(\det(sI-(F-LH))} = s^2 + (4+\ell_2+1+\ell_1)s + 4 + \ell_2 + 4\ell_1 + \cancel{\ell_1\ell_2} - \cancel{\ell_1\ell_2}$$

$\alpha_e(s) = (s+40+40j)(s+40-40j) = s^2 + 80s + 3200$

ENTÃO: $s^2 + (5+\ell_1+\ell_2)s + 4 + 4\ell_1 + \ell_2 = s^2 + 80s + 3200$

$$\begin{matrix} \ell_1 + \ell_2 = 75 \\ 4\ell_1 + \ell_2 = 3196 \end{matrix} \longrightarrow \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} = \frac{\begin{bmatrix} 1 & -1 \\ -4 & 1 \end{bmatrix}}{-3} \begin{bmatrix} 75 \\ 3196 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} = \begin{bmatrix} 1040.33 \\ -965.33 \end{bmatrix}}$$

b) TRANSFORMAÇÃO PARA A FORMA CANÔNICA OBSERVÁVEL : $x = Tz$

$\mathcal{O} = \begin{bmatrix} 1 & 1 \\ -1 & -4 \end{bmatrix}$ $\quad \mathcal{O}^{-1} = \frac{\begin{bmatrix} 4 & 1 \\ -1 & -1 \end{bmatrix}}{3}$ $\quad t_2 = \mathcal{O}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$

$t_1 = F t_2 = \begin{bmatrix} -\frac{1}{3} \\ \frac{4}{3} \end{bmatrix}$

ENTÃO: $T = \frac{1}{3} \begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix} \longrightarrow T^{-1} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$

$F_0 = T^{-1} F T = \frac{1}{3} \begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ -4 & 0 \end{bmatrix}$

$H_0 = HT = [1 \quad 1] \frac{\begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix}}{3} = [1 \quad 0]$

OBS.: VERIFICAÇÃO DE RESULTADOS QUE JÁ ERAM ESPERADOS.

$F_0 - L_0 H_0 = \begin{bmatrix} -5 & 1 \\ -4 & 0 \end{bmatrix} - \begin{bmatrix} \ell_{10} \\ \ell_{20} \end{bmatrix} [1 \quad 0] = \begin{bmatrix} -5-\ell_{10} & 1 \\ -4-\ell_{20} & 0 \end{bmatrix}$

$s^2 + (\ell_{10}+5)s + (\ell_{20}+4) = s^2 + 80s + 3200$

$\ell_{10} = 75$

$\ell_{20} = 3196$

MUDANÇA PARA O SISTEMA DE COORDENADAS ORIGINAL:

$$\hat{z}' = F_0\hat{z} + G_0 u + L_0(y - H_0\hat{z}) \longrightarrow \text{F. OBSERVÁVEL}$$

$$\downarrow \ \hat{z} = T^{-1}\hat{x}$$

$$T^{-1}\hat{x}' = F_0 T^{-1}\hat{x} + G_0 u + L_0(y - H_0 T^{-1}\hat{x})$$

$$\hat{x}' = TF_0T^{-1}\hat{x} + TG_0 u + TL_0(y - H_0T^{-1}\hat{x})$$

$$\hat{x}' = F\hat{x} + Gu + L(y - H\hat{x}) \longrightarrow \text{F. ORIGINAL}$$

ENTÃO: $L = TL_0 = \frac{1}{3}\begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 75 \\ 3196 \end{bmatrix} \longrightarrow \boxed{L = \begin{bmatrix} 1040.33 \\ -965.33 \end{bmatrix}}$

c) $\alpha_e(s) = (s + 40 + 40j)(s + 40 - 40j) = s^2 + 80s + 3200$

$$\alpha_e(F) = \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}^2 + 80\begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix} + 3200\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3121 & 0 \\ 0 & 2896 \end{bmatrix}$$

$$L = \begin{bmatrix} 3121 & 0 \\ 0 & 2896 \end{bmatrix} \underbrace{\frac{1}{3}\begin{bmatrix} 4 & 1 \\ -1 & -1 \end{bmatrix}}_{\mathcal{O}^{-1}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3121 \\ -2896 \end{bmatrix}$$

$$\boxed{L = \begin{bmatrix} 1040.33 \\ -965.33 \end{bmatrix}}$$

EXERCICIO #6

a) CAPACITOR : $i_c = \frac{dq}{dt} = C\dot{V} \qquad (q = CV)$

$$\underbrace{R_1 i_c}_{C\dot{V} = Cx_1'} + \underbrace{V}_{x_1} = u$$

ENTÃO : $x_1' = -\frac{x_1}{R_1 C} + \frac{u}{R_1 C}$

INDUTOR : $v = L\frac{di_L}{dt}$

$$\underbrace{R_2 i_L}_{x_2} + L\underbrace{\frac{di_L}{dt}}_{x_2'} = u$$

ENTÃO : $x_2' = -\frac{R_2}{L}x_2 + \frac{u}{L}$

SAÍDA : $y = \frac{u - x_1}{R_1} + x_2$

EQUAÇÕES DE ESTADO :

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \overbrace{\begin{bmatrix} -\frac{1}{R_1 C} & 0 \\ 0 & -\frac{R_2}{L} \end{bmatrix}}^{F} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \overbrace{\begin{bmatrix} \frac{1}{R_1 C} \\ \frac{1}{L} \end{bmatrix}}^{G} u$$

$$y = \underbrace{\begin{bmatrix} -\frac{1}{R_1} & 1 \end{bmatrix}}_{H} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\frac{1}{R_1}}_{J} u$$

b) CONTROLABILIDADE

$$\mathcal{C} = \begin{bmatrix} G & FG \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1 C} & \frac{-1}{R_1^2 C^2} \\ \frac{1}{L} & -\frac{R_2}{L^2} \end{bmatrix}$$

$$\det \mathcal{C} = \frac{-R_2}{R_1 C L^2} + \frac{1}{R_1^2 C^2 L} = 0$$

$$\frac{R_2}{L} = \frac{1}{R_1 C} \longrightarrow$$ SE $\frac{R_2}{L} = \frac{1}{R_1 C}$ (MESMAS CONSTANTES DE TEMPO), O SISTEMA NÃO É CONTROLÁVEL.

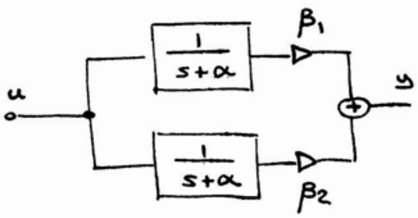
OBSERVABILIDADE :

$$\mathcal{O} = \begin{bmatrix} H \\ HF \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1} & 1 \\ \frac{1}{R_1^2 C} & -\frac{R_2}{L} \end{bmatrix}$$

$$\det \Theta = \frac{R_2}{R_1 L} - \frac{1}{R_1^2 C} = 0$$

$$\frac{R_2}{L} = \frac{1}{R_1 C} \longrightarrow$$ SE $\frac{R_2}{L} = \frac{1}{R_1 C}$, O SISTEMA É TAMBÉM NÃO-OBSERVÁVEL.

c) SE AS CONSTANTES DE TEMPO τ SÃO IGUAIS, O SISTEMA COMPORTA-SE COMO A ASSOCIAÇÃO EM PARALELO DE DUAS FUNÇÕES DE TRANSFERÊNCIA IGUAIS:



SISTEMAS DESTE TIPO SÃO NÃO-CONTROLÁVEIS E NÃO-OBSERVÁVEIS (BASTA ESCREVER AS EQUAÇÕES DE ESTADO PARA VERIFICAR).

d) $$\frac{Y(s)}{U(s)} = H(sI-F)^{-1}G + J$$

$$\frac{Y(s)}{U(s)} = \frac{\begin{bmatrix} -\frac{1}{R_1} & 1 \end{bmatrix} \begin{bmatrix} s+\frac{R_2}{L} & 0 \\ 0 & s+\frac{1}{R_1 C} \end{bmatrix}}{\left(s+\frac{R_2}{L}\right)\left(s+\frac{1}{R_1 C}\right)} \begin{bmatrix} \frac{1}{R_1 C} \\ \frac{1}{L} \end{bmatrix} + \frac{1}{R_1}$$

$$= \frac{-\frac{1}{R_1}\left(s+\frac{R_2}{L}\right)\frac{1}{R_1 C} + \left(s+\frac{1}{R_1 C}\right)\frac{1}{L} + \frac{1}{R_1}\left(s+\frac{R_2}{L}\right)\left(s+\frac{1}{R_1 C}\right)}{\left(s+\frac{R_2}{L}\right)\left(s+\frac{1}{R_1 C}\right)}$$

$$= \frac{\frac{1}{R_1}\left(s^2 + \left(\frac{R_1+R_2}{L}\right)s + \frac{1}{LC}\right)}{\left(s+\frac{R_2}{L}\right)\left(s+\frac{1}{R_1 C}\right)}$$

SE $\frac{L}{R_2} = R_1 C$:

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{R_1}\left(s^2 + \left(\frac{R_1}{L} + \frac{1}{R_1 C}\right)s + \frac{1}{LC}\right)}{\left(s+\frac{1}{R_1 C}\right)^2}$$

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{R_1}\cancel{\left(s+\frac{1}{R_1 C}\right)}\left(s+\frac{R_1}{L}\right)}{\left(s+\frac{1}{R_1 C}\right)\cancel{\left(s+\frac{1}{R_1 C}\right)}}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{R_1} \cdot \frac{\left(s+\frac{R_1}{L}\right)}{\left(s+\frac{1}{R_1 C}\right)}$$

HOUVE CANCELAMENTO ENTRE UM POLO E UM ZERO EM $s = -\frac{1}{R_1 C}$.