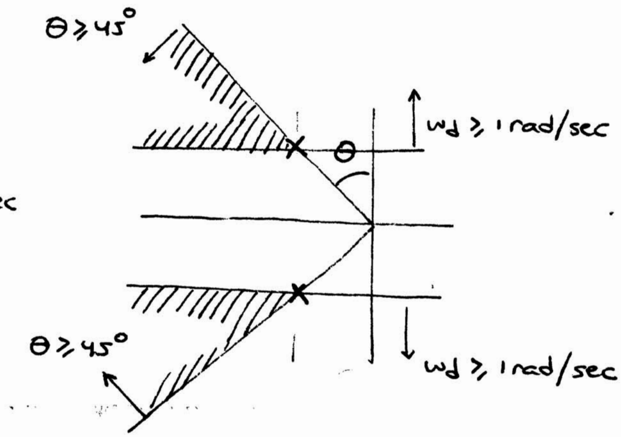
CONTROLE LINEAR II - PRIMEIRO SEMESTRE DE 2006 - GABARITO DA LISTA DE EXERCICIOS #3

EXERCICIO #1

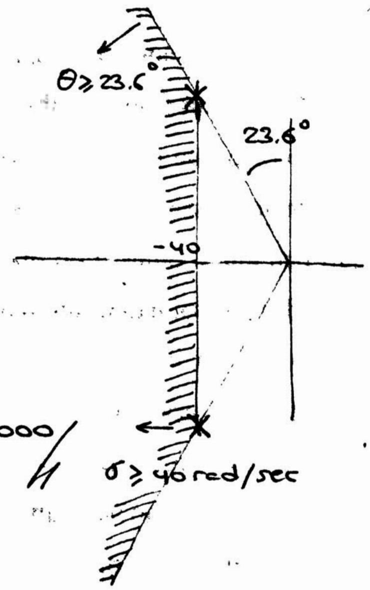


a) $\xi \geq 0.707 \longrightarrow \theta \geq 45°$

$t_p = \dfrac{\pi}{\omega_d} \leq 3.14$ sec $\longrightarrow \omega_d \geq 1$ rad/sec

$s = -1 \pm j$

$\alpha_c(s) = (s+1+j)(s+1-j) = s^2 + 2s + 2$ //



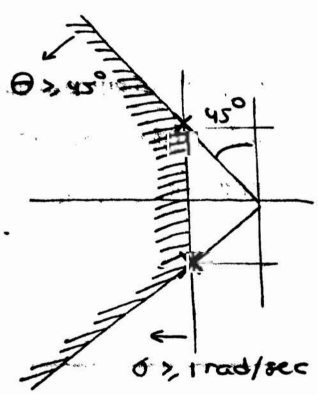
b) $M_p \leq 25\% \longrightarrow \xi \geq 0.4 \; (\theta \geq 23.6°)$

$\dfrac{4.6}{\sigma} = t_s \leq 0.115 \longrightarrow \sigma \geq \dfrac{4.6}{0.115} = 40$ rad/sec

$\dfrac{\sigma}{\omega_d} = \tan\theta = 0.436 \longrightarrow \omega_d = 92$ rad/sec

$s = -40 \pm 92j$

$\alpha_c(s) = (s+40+92j)(s+40-92j) = s^2 + 80s + 10000$ //



c) $M_p \leq 5\% \longrightarrow \xi \geq 0.707 \longrightarrow \theta \geq 45°$

$\dfrac{4.6}{\sigma} = t_s \leq 4.6 \longrightarrow \sigma \geq 1$ rad/sec

$s_{1,2} = -1 \pm j$

$s_3 = -4$

$\alpha_c(s) = \underbrace{(s+1+j)(s+1-j)}_{s^2+2s+2}(s+4)$

$\alpha_c(s) = s^3 + 6s^2 + 10s + 8$ //

EXERCICIO #2

Verificação dos resultados do problema #1 no MATLAB

```
>> figure(1); sys=tf(2,[1 2 2]); step(sys); grid on;
>> figure(2); sys=tf(10000,[1 80 10000]); step(sys); grid on;
>> figure(3); sys=tf(8,[1 6 10 8]); step(sys); grid on;
```

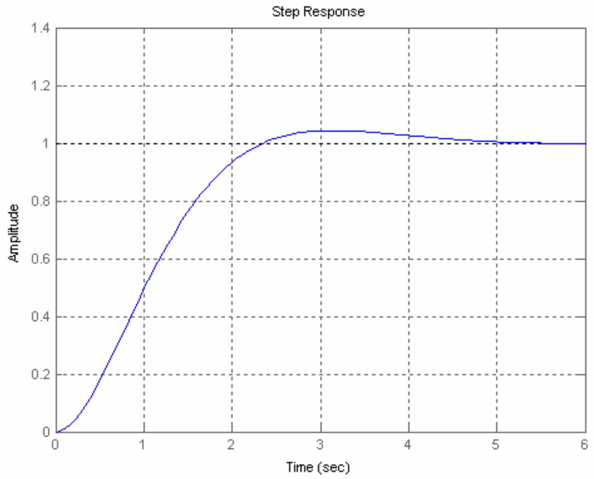
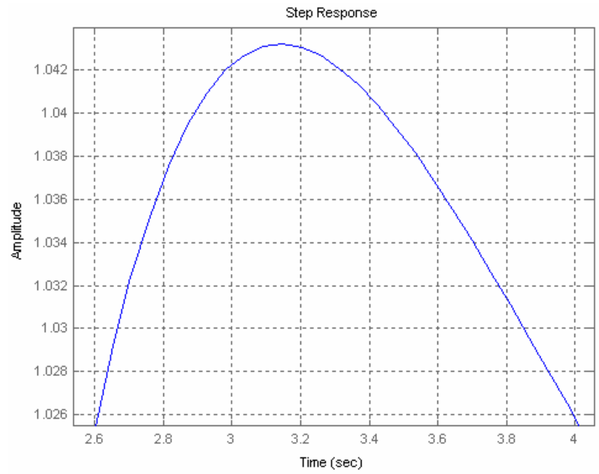


Figura 1



Detalhe da Figura 1

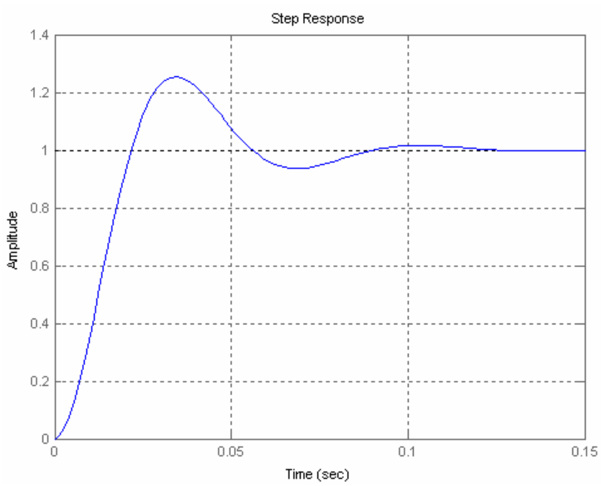
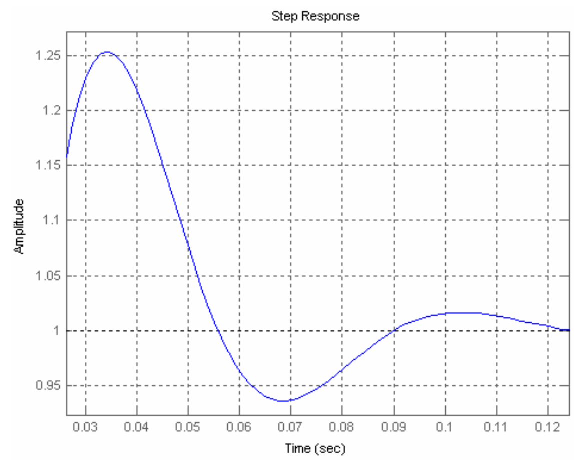


Figura 2



Detalhe da Figura 2

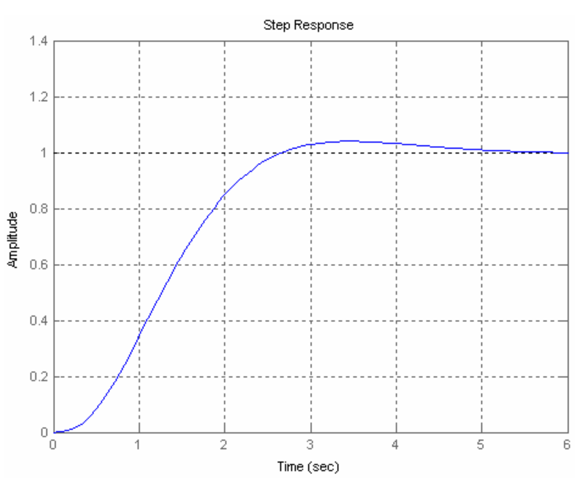
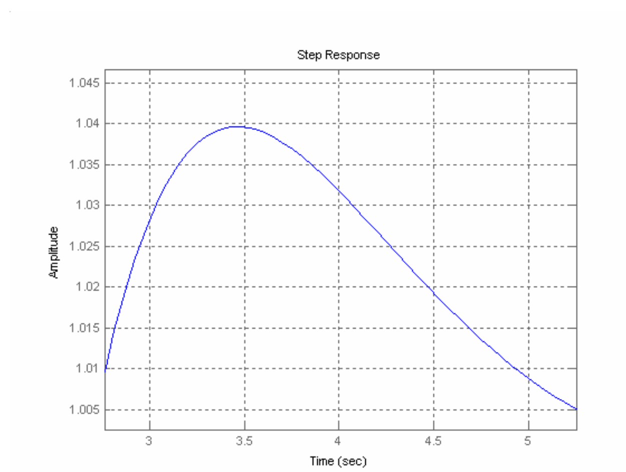


Figura 3



Detalhe da Figura 3

Obs: Todos os sistemas alcançam resultados bem próximos das especificações. Para melhorar os resultados, uma possibilidade é mover todos os pólos ligeiramente para a esquerda (isso reduz o tempo de estabelecimento e aumenta o fator de amortecimento).

EXERCÍCIO #3

$$\underbrace{\begin{bmatrix}\theta\\ \dot\theta\\ x\\ \dot x\end{bmatrix}'}_{\text{(DERIVADO)}} = \begin{bmatrix}\dot\theta\\ \ddot\theta\\ \dot x\\ \ddot x\end{bmatrix} = \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\-\beta&0&0&0\end{bmatrix}\begin{bmatrix}\theta\\ \dot\theta\\ x\\ \dot x\end{bmatrix} + \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}u$$

$$\alpha_c(s) = (s+1)^2(s+1-j)(s+1+j) = (s^2+2s+1)(s^2+2s+2) = s^4+4s^3+7s^2+6s+2$$

$$F-GK = \begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\-\beta&0&0&0\end{bmatrix} - \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}[k_1\ \ k_2\ \ k_3\ \ k_4] = \begin{bmatrix}0&1&0&0\\1-k_1&-k_2&-k_3&-k_4\\0&0&0&0\\-\beta+k_1&k_2&k_3&k_4\end{bmatrix}.$$

$$|sI-F+GK| = \begin{vmatrix}s&-1&0&0\\k_1-1&s+k_2&k_3&k_4\\0&0&s&-1\\\beta-k_1&-k_2&-k_3&s-k_4\end{vmatrix}$$

$$= s\underbrace{\begin{vmatrix}s+k_2&k_3&k_4\\0&s&-1\\-k_2&-k_3&s-k_4\end{vmatrix}}_{s^3+(k_2-k_4)s^2-k_3s} + 1\cdot\underbrace{\begin{vmatrix}k_1-1&k_3&k_4\\0&s&-1\\\beta-k_4&-k_3&s-k_4\end{vmatrix}}_{(k_1-1)s^2+(k_4-\beta k_4)s+(k_3-\beta k_3)}$$

$$|sI-F+GK| = s^4+(k_2-k_4)s^3+(k_1-k_3-1)s^2+[(1-\beta)k_4]s+(1-\beta)k_3$$

COMPARANDO COM COEFICIENTES DE $\alpha_c(s)$:

$$k_3 = \frac{2}{1-\beta} \qquad k_4 = \frac{6}{1-\beta}$$

$$k_1 = 8+k_3 \qquad k_2 = 4+k_4 = \frac{10-4\beta}{1-\beta}$$

$$k_1 = \frac{10-8\beta}{1-\beta}$$

$$K = \begin{bmatrix}\frac{10-8\beta}{1-\beta} & \frac{10-4\beta}{1-\beta} & \frac{2}{1-\beta} & \frac{6}{1-\beta}\end{bmatrix}$$

EXERCICIO #4

a) $\mathcal{O} = \begin{bmatrix}H\\HF\end{bmatrix} = \begin{bmatrix}1&2\\0&1\end{bmatrix}$ $\qquad \det\mathcal{O} = 1 \neq 0 \qquad$ ((F,H) É OBSERVÁVEL)

* b) $x' = (F-GK)x$

$y = Hx$

$$F-GK = \begin{bmatrix}-2&1\\1&0\end{bmatrix} - \begin{bmatrix}1\\0\end{bmatrix}[k_1\ \ k_2] = \begin{bmatrix}-2-k_1&1-k_2\\1&0\end{bmatrix}$$

PARA O SISTEMA EM MALHA FECHADA $((F-GK, H))$:

$$\mathcal{O} = \begin{bmatrix}H\\H(F-GK)\end{bmatrix} = \begin{bmatrix}1&2\\-k_1&1-k_2\end{bmatrix}$$

$$\det\mathcal{O} = 1-k_2+2k_1$$

SE $\boxed{2k_1-k_2+1=0}$ $\longrightarrow$ SISTEMA $(F-GK, H)$ NÃO-OBSERVÁVEL.

c) $k_1 = 1 \quad e \quad 2k_1 - k_2 + 1 = 0 \implies k_2 = 3$

$$\boxed{K = [1 \;\; 3]}$$

d) FUNÇÃO DE TRANSFERÊNCIA EM MALHA ABERTA $\left(G(s) = \dfrac{Y(s)}{U(s)} \right)$

$x' = Fx + Gu$

$y = Hx$

$$\frac{Y(s)}{U(s)} = H(sI - F)^{-1}G = [1 \;\; 2] \begin{bmatrix} s+2 & -1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{[1 \;\; 2] \begin{bmatrix} s & 1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2 + 2s - 1} \longrightarrow \boxed{\frac{Y(s)}{U(s)} = \frac{s+2}{s^2 + 2s - 1}}$$

FUNÇÃO DE TRANSFERÊNCIA EM MALHA FECHADA $\left(u = -Kx + \bar{N}r \;;\; \dfrac{Y(s)}{R(s)} \right)$

$x' = (F - GK)x + G\bar{N}r$

$y = Hx$

$$\frac{Y(s)}{R(s)} = H(sI - (F - GK))^{-1} G\bar{N}$$

$$F - GK \Big|_{K = [1 \;\; 3]} = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}$$

$$sI - (F - GK) = \begin{bmatrix} s+3 & 2 \\ -1 & s \end{bmatrix} \qquad (sI - (F - GK))^{-1} = \frac{\begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}}{s^2 + 3s + 2}$$

$$\frac{Y(s)}{R(s)} = \frac{[1 \;\; 2] \begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}}{s^2 + 3s + 2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bar{N}$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{\cancel{(s+2)}\,\bar{N}}{(s+1)\cancel{(s+2)}} = \frac{\bar{N}}{s+1}}$$

O SISTEMA EM MALHA FECHADA NÃO É OBSERVÁVEL, POR CAUSA DO CANCELAMENTO ENTRE UM PÓLO E UM ZERO EM $s = -2$.

EXERCICIO #5

a)
$$\begin{vmatrix} s+0.4 & 0 & 0.01 \\ -1 & s & 0 \\ 1.4 & -9.8 & s+0.02 \end{vmatrix} = \frac{1}{100}\left(100s^3 + 42s^2 - 0.6s + 9.8\right) = 0$$

PÓLOS (RAÍZES CALCULADOS NUMERICAMENTE NO MATLAB):

$$\boxed{s_1 = -0.6565 \quad ; \quad s_2, s_3 = 0.1183 \pm 0.3678j}$$

b)
$$\mathcal{C} = [\,G \quad FG \quad F^2G\,] = \begin{bmatrix} 6.3 & -2.618 & 1.1374 \\ 0 & 6.3 & -2.618 \\ 9.8 & -9.016 & 65.5855 \end{bmatrix}$$

$$\det \mathcal{C} = 2.4513 \times 10^3 \longrightarrow \text{SISTEMA CONTROLÁVEL}$$

c)
$$\alpha_c(s) = (s+2)\underbrace{(s+1+j)(s+1-j)}_{s^2+2s+2} = s^3 + 4s^2 + 6s + 4$$

$$\alpha_c(F) = \begin{bmatrix} 2.12252 & -0.35084 & -0.04502 \\ 4.574 & 3.902 & -0.03580 \\ 28.7806 & 58.1571 & 3.83343 \end{bmatrix}$$

$$\mathcal{C}^{-1} = \begin{bmatrix} 0.15893 & 0.06586 & -0.00013 \\ -0.01047 & 0.16401 & 0.00673 \\ -0.02519 & 0.01271 & 0.01619 \end{bmatrix}$$

$$K = [\,0 \quad 0 \quad 1\,]\, \mathcal{C}^{-1} \alpha_c(F)$$

$$\boxed{K = [\,0.47065 \quad 1 \quad 0.06275\,]}$$