EEL760 — Lista de Exercícios #1 — Gabarito

1) a) FCC: $F = \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad H = [1 \;\; 1]$ $\quad J = 0$

FCO: $F = \begin{bmatrix} -5 & 1 \\ -6 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad H = [1 \;\; 0]$ $\quad J = 0$

FCM: $G(s) = \dfrac{A}{s+2} + \dfrac{B}{s+3} = \dfrac{-1}{s+2} + \dfrac{2}{s+3} \longrightarrow F = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \quad G = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$A = \dfrac{-1}{1} = -1 \qquad B = \dfrac{-2}{-1} = 2 \qquad H = [1 \;\; 1] \quad J = 0$

b) FCC: $F = \begin{bmatrix} -3 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\quad H = [0 \;\; 1 \;\; 3]$ $\quad J = 0$

FCO: $F = \begin{bmatrix} -3 & 1 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ $\quad H = [1 \;\; 0 \;\; 0]$ $\quad J = 0$

FCM: $G(s) = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{s+2} = \dfrac{3/2}{s} + \dfrac{-2}{s+1} + \dfrac{1/2}{s+2} \longrightarrow F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad G = \begin{bmatrix} 3/2 \\ -2 \\ 1/2 \end{bmatrix}$

$A = \dfrac{3}{2} \qquad B = \dfrac{2}{-1} = -2 \qquad C = \dfrac{1}{2} \qquad H = [1 \;\; 1 \;\; 1] \quad J = 0$

c) FCC: $F = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad H = [1 \;\; 1]$ $\quad J = 0$ $\quad$ (NÃO-OBSERVÁVEL)

FCO: $F = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$ $\quad G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad H = [1 \;\; 0]$ $\quad J = 0$ $\quad$ (NÃO-CONTROLÁVEL)

FCM: $G(s) = \dfrac{A}{s+1} + \dfrac{B}{s+2} = \dfrac{0}{s+1} + \dfrac{1}{s+2}$

$A = 0 \qquad B = \dfrac{-1}{-1} = 1$

$F = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ $\quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad H = [1 \;\; 1]$ $\quad J = 0$

NOTE QUE ESTA REPRESENTAÇÃO NÃO É CONTROLÁVEL. PODERÍAMOS ESCOLHER UMA QUE FOSSE.

d) FCC: $F = -2 \quad G = 1 \quad H = 1 \quad J = 0$

FCM: $F = -2 \quad G = 1 \quad H = 1 \quad J = 0$

FCO: $F = -2 \quad G = 1 \quad H = 1 \quad J = 0$

2) a) $\mathcal{O} = \begin{bmatrix} H_{cc} \\ H_{cc} F_{cc} \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & -2 \end{bmatrix}$

$$t_2 = \mathcal{O}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{\begin{bmatrix} -2 & -3 \\ -1 & 1 \end{bmatrix}}{-5} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/5 \\ -1/5 \end{bmatrix}$$

$$t_1 = F_{cc} t_2 = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} \frac{1}{5} = \begin{bmatrix} -4/5 \\ 3/5 \end{bmatrix}$$

$$T = \frac{1}{5} \begin{bmatrix} -4 & 3 \\ 3 & -1 \end{bmatrix} \quad ; \quad T^{-1} = \begin{bmatrix} -1/5 & -3/5 \\ -3/5 & -4/5 \end{bmatrix} \times \left(\frac{-1}{1/5} \right) = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

b) $$F_{co} = T^{-1} F_{cc} T = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -4 & 3 \\ 3 & -1 \end{bmatrix} \cdot \frac{1}{5} = \begin{bmatrix} 1 & -2 \\ -2 & -6 \end{bmatrix} \begin{bmatrix} -4 & 3 \\ 3 & -1 \end{bmatrix} \cdot \frac{1}{5} = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}$$

$$G_{co} = T^{-1} G_{cc} = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$H_{co} = H_{cc}T = [1 \quad 3] \begin{bmatrix} -4 & 3 \\ 3 & -1 \end{bmatrix} \cdot \frac{1}{5} = [1 \quad 0]$

$J_{co} = 1$

NOTE QUE: $F_{co} = F_{cc}^T$, $G_{co} = H_{cc}^T$ E $H_{co} = G_{cc}^T$

c) $G(s) = H(sI - F)^{-1}G + J$

USANDO A F_{cc}:

$$G(s) = [1 \quad 3] \begin{bmatrix} s+2 & 2 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1 = \frac{[1 \quad 3] \begin{bmatrix} s & -2 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2 + 2s + 2} + 1$$

$$= \frac{s + 3 + s^2 + 2s + 2}{s^2 + 2s + 2} = \frac{s^2 + 3s + 5}{s^2 + 2s + 2}$$

OPCIONALMENTE, USANDO A F_{co}:

$$G(s) = [1 \quad 0] \begin{bmatrix} s+2 & -1 \\ 2 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + 1 = [1 \quad 0] \begin{bmatrix} s & 1 \\ -2 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + 1$$

$$= \frac{s+3}{s^2 + 2s + 2} + 1 = \frac{s^2 + 3s + 5}{s^2 + 2s + 2}$$

3) $x = Tz$, ONDE $\det T \neq 0$

ALÉM DISSO, $\mathcal{O} = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{n-1} \end{bmatrix}$, ONDE $\det \mathcal{O} \neq 0$ $\left(\text{SENDO } \begin{matrix} x' = Fx + Gu \\ y = Hx + Ju \end{matrix} \right)$

ENTÃO:

$$\mathcal{O}_z = \begin{bmatrix} H_z \\ H_z F_z \\ \vdots \\ H_z F_z^{n-1} \end{bmatrix} = \begin{bmatrix} HT \\ HTT^{-1}FT \\ \vdots \\ HT(T^{-1}FT)^{n-1} \end{bmatrix} = \begin{bmatrix} HT \\ HFT \\ \vdots \\ HF^{n-1}T \end{bmatrix} = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{n-1} \end{bmatrix} T = \mathcal{O}T$$

PORTANTO: $\det \mathcal{O} \neq 0$ E $\det T \neq 0 \iff \det \mathcal{O}_z \neq 0$: UMA TRANSFORMAÇÃO LINEAR NÃO-SINGULAR TRANSFORMA UM SISTEMA OBSERVÁVEL EM OUTRO SISTEMA OBSERVÁVEL.

4) $\mathcal{C} = [G \quad FG] = \begin{bmatrix} 1 & -3+\beta \\ \beta & -2 \end{bmatrix}$

SISTEMA NÃO-CONTROLÁVEL $\iff \det \mathcal{C} = 0$

$$\begin{vmatrix} 1 & -3+\beta \\ \beta & -2 \end{vmatrix} = -2 - 3\beta - \beta^2 = 0$$

$$\beta^2 + 3\beta + 2 = 0$$

$$\beta = \frac{-3 \pm \sqrt{9-8}}{2} \longrightarrow \beta = -1 \text{ OU } \beta = -2$$

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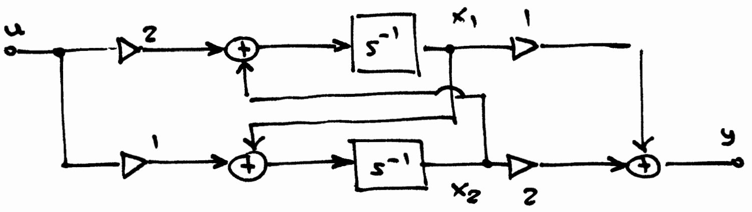
5) a) $x_1' = x_2 + 2u$

$x_2' = x_1 + u$

$y = x_1 + 2x_2$



b) $$G(s) = [1 \quad 2]\begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{[1 \quad 2]\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}}{s^2-1} = \frac{[1 \quad 2]\begin{bmatrix} 2s+1 \\ s+2 \end{bmatrix}}{s^2-1} = \frac{4s+5}{s^2-1}$$

c) $$Y(s) = (H(sI-F)^{-1}G + J)U(s) + \underbrace{H(sI-F)^{-1}x(0)}_{0}$$

$$Y(s) = \frac{4s+5}{s^2-1}\cdot\frac{1}{s} = \frac{4s+5}{s(s+1)(s-1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} = \frac{-5}{s} + \frac{1/2}{s+1} + \frac{9/2}{s-1}$$

$A = \frac{-5}{1} = -5 \qquad B = \frac{1}{2} \qquad C = \frac{9}{2}$

TRANSFORMADA DE LAPLACE INVERSA:

$$y(t) = \left(-5 + \frac{1}{2}e^{-t} + \frac{9}{2}e^{t}\right)u(t)$$

d) $$Y(s) = \frac{4s+5}{s^2-1}\cdot\frac{1}{s} + \frac{[1 \quad 2]\begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2-1} = ?$$

$$= \frac{4s+5}{s(s^2-1)} + \frac{3s+3}{s^2-1} = \frac{3s^2+7s+5}{s(s+1)(s-1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1}$$

$A = \frac{-5}{1} = -5 \qquad B = \frac{1}{2} \qquad C = \frac{15}{2}$

$$Y(s) = \frac{-5}{s} + \frac{1/2}{s+1} + \frac{15/2}{s-1}$$

TRANSFORMADA DE LAPLACE INVERSA:

$$y(t) = \left(-5 + \frac{1}{2}e^{-t} + \frac{15}{2}e^{t}\right)u(t)$$

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SUGESTÃO: VERIFIQUE OS RESULTADOS DOS ITENS 5c E 5d NO MATLAB!

EXECUTE OS COMANDOS:

```
sys = ss([0 1; 1 0],[2; 1],[1 2],0);
a=0; b=5; pts = 10000; stp=(b-a)/pts; t=a:stp:(b-stp);
u = ones(size(t));
lsim(sys,u,t); hold on;
plot(t,-5+0.5*exp(-t)+4.5*exp(t),'r-');
figure;
lsim(sys,u,t,[1; 1]); hold on;
plot(t,-5+0.5*exp(-t)+7.5*exp(t),'r-');
```