Lista de Exercícios #11 — Controle ~~X~~ II — Gabarito

1) $\Phi = \begin{bmatrix} 1.8 & 1 \\ -0.81 & 0 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} \qquad H = [1 \quad 0]$

a) $\Phi - LH = \begin{bmatrix} 1.8 & 1 \\ -0.81 & 0 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$

$\cancel{\alpha} \; \alpha_e(z) = (z-0.1)(z-0.2) = z^2 - 0.3z + 0.02$

$|zI - \Phi + LH| = \begin{vmatrix} z - 1.8 + l_1 & -1 \\ +0.81 + l_2 & z \end{vmatrix}$

$-1.8 + l_1 = -0.3 \longrightarrow l_1 = 1.5$

$0.81 + l_2 = 0.02 \longrightarrow l_2 = -0.79$

$L = \begin{bmatrix} 1.5 \\ -0.79 \end{bmatrix}$

b) $\Phi - LH\Phi = \begin{bmatrix} 1.8 & 1 \\ -0.81 & 0 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 1.8 & 1 \end{bmatrix} = \begin{bmatrix} 1.8 - 1.8 l_1 & 1 - l_1 \\ -0.81 - 1.8 l_2 & -l_2 \end{bmatrix}$

$|zI - \Phi + LH\Phi| = \begin{vmatrix} z - 1.8 + 1.8 l_1 & -1 + l_1 \\ 0.81 + 1.8 l_2 & z + l_2 \end{vmatrix} = z^2 + (-1.8 + 1.8 l_1 + l_2)z \cancel{- 1.8 l_2} \cancel{+ 1.8 l_1 l_2} + 0.81 \cancel{+ 1.8 l_2}$
$- 0.81 l_1 \cancel{- 1.8 l_1 l_2}$

$= z^2 + (1.8 l_1 + l_2 - 1.8)z + 0.81 - 0.81 l_1$

$0.81 l_1 = 0.79$

$l_1 = \frac{79}{81}$

$1.8 \times \frac{79}{81} - 1.8 + l_2 = -0.3$

$l_2 = -0.3 + \frac{0.4}{9} = -\frac{2.3}{9} = -\frac{23}{90}$

$L = \begin{bmatrix} 79/81 \\ -23/90 \end{bmatrix}$

Note que $\begin{bmatrix} 79/81 \\ -23/90 \end{bmatrix} = \Phi^{-1} \begin{bmatrix} 1.5 \\ -0.79 \end{bmatrix}$

$\Phi - LH = \begin{bmatrix} 0.3 & 1 \\ -0.02 & 0 \end{bmatrix}$

c) Estimador de predição:

(k-1): $u(k-1) \longrightarrow$ conversor D/A

conversor A/D $\longrightarrow y(k-1)$

$$\begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} = \begin{bmatrix} 0.3 & 1 \\ -0.02 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k-1) \\ \hat{x}_2(k-1) \end{bmatrix} + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(k-1) + \begin{bmatrix} 1.5 \\ -0.79 \end{bmatrix} y(k-1)$$

$u(k) = -k_1 \hat{x}_1(k) - k_2 \hat{x}_2(k)$

~~X~~ Espero

(k): $u(k) \longrightarrow$ conversor D/A

conversor A/D $\longrightarrow$ ~~[illegible]~~ $y(k)$

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.3 & 1 \\ -0.02 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(k) + \begin{bmatrix} 1.5 \\ -0.79 \end{bmatrix} y(k)$$

$u(k+1) = -k \hat{x}_1(k+1) - k_2 \hat{x}_2(k+1)$

~~X~~ Espero

... e assim por diante.

ESTIMADOR ATUAL:

DISPONÍVEL DO ITERAÇÃO ~~k-1~~ k-2

(k-1): CONVERSOR A/D → ~~y(k)~~ y(k-1)

$$\begin{bmatrix}\hat{x}_1(k-1)\\ \hat{x}_2(k-1)\end{bmatrix} = \begin{bmatrix}\check{x}_1(k-1)\\ \check{x}_2(k-1)\end{bmatrix} + \begin{bmatrix}79/81\\ -23/90\end{bmatrix} y(k-1)$$

$$u(k-1) = -K_1 x_1(k-1) - K_2 x_2(k-1)$$

u(k-1) ——→ CONVERSOR D/A

$$\cancel{\begin{bmatrix}\check{x}_1(k)\\ \check{x}_2(k)\end{bmatrix} = \begin{bmatrix}0.3 & 1\\ -0.02 & 0\end{bmatrix}\begin{bmatrix}\hat{x}_1(k-1)\\ \hat{x}_2(k-1)\end{bmatrix}}$$

$\Phi - LH\Phi$ $\qquad\qquad$ $\Gamma - LH\Gamma$

$$\begin{bmatrix}\check{x}_1(k)\\ \check{x}_2(k)\end{bmatrix} = \begin{bmatrix}\cancel{0.3} & 0.0444 & 0.0247\\ \cancel{-0.02} & -0.35 & 0.2556\end{bmatrix}\begin{bmatrix}\hat{x}_1(k-1)\\ \hat{x}_2(k-1)\end{bmatrix} + \begin{bmatrix}0.0247\\ -0.2444\end{bmatrix} u(k-1)$$

~~Δt~~ ESPERO

(k): CONVERSOR A/D → y(k)

$$\begin{bmatrix}\hat{x}_1(k)\\ \hat{x}_2(k)\end{bmatrix} = \begin{bmatrix}\check{x}_1(k)\\ \check{x}_2(k)\end{bmatrix} + \begin{bmatrix}79/81\\ -23/90\end{bmatrix} y(k)$$

$$u(k) = -K_1 \hat{x}_1(k) - K_2 \hat{x}_2(k)$$

u(k) ——→ CONVERSOR D/A

$$\begin{bmatrix}\check{x}_1(k+1)\\ \check{x}_2(k+1)\end{bmatrix} = \begin{bmatrix}0.0444 & 0.0247\\ -0.35 & 0.2556\end{bmatrix}\begin{bmatrix}\hat{x}_1(k)\\ \hat{x}_2(k)\end{bmatrix} + \begin{bmatrix}0.0247\\ -0.2444\end{bmatrix} u(k)$$

~~Δt~~ ESPERO

... E ASSIM POR DIANTE.

OBS.: PARA O ESTIMADOR ATUAL:

$$\hat{x}(k+1) = \underbrace{(\Phi - LH\Phi)\hat{x}(k) + (\Gamma - LH\Gamma)u(k)}_{\check{x}(k+1)} + Ly(k+1)$$

$\check{x}(k+1)$ (TERMO PARCIAL QUE PODE SER CALCULADO NO INSTANTE k).

② $\Phi = \begin{bmatrix}1.8 & 1\\ -0.81 & 0\end{bmatrix}$ $\quad \Gamma = \begin{bmatrix}1\\ -0.85\end{bmatrix}$ $\quad H = [1 \;\; 0]$

a) $L = \begin{bmatrix}1.5\\ -0.79\end{bmatrix}$ NOTE QUE L NÃO DEPENDE DE Γ.

b) $L = \begin{bmatrix}79/81\\ -23/90\end{bmatrix}$ NOTE QUE L NÃO DEPENDE DE Γ.

c) OS PROGRAMAS PARA O ESTIMADOR DE PREDIÇÃO E PARA O ESTIMADOR ATUAL CONTINUAM SENDO OS MESMOS DE ANTES. AS ÚNICAS MODIFICAÇÕES A SEREM FEITAS SÃO:

- PROGRAMA DO ESTIMADOR DE PREDIÇÃO: SUBSTITUIR $\Gamma = \begin{bmatrix}1\\ -0.5\end{bmatrix}$ POR $\Gamma = \begin{bmatrix}1\\ -0.85\end{bmatrix}$
- PROGRAMA DO ESTIMADOR ATUAL: SUBSTITUIR $\Gamma - LH\Gamma = \begin{bmatrix}0.0247\\ -0.2444\end{bmatrix}$ POR $\Gamma - LH\Gamma = \begin{bmatrix}0.0247\\ -0.5944\end{bmatrix}$

③ ~~F~~ $\Phi = \begin{bmatrix}-1 & -0.5\\ 1 & 0\end{bmatrix}$ $\quad \Gamma = \begin{bmatrix}1\\ 0\end{bmatrix}$ $\quad H = [1 \;\; -0.5]$

a) $\Phi - \Gamma K = \begin{bmatrix}-1 & -0.5\\ 1 & 0\end{bmatrix} - \begin{bmatrix}1\\ 0\end{bmatrix}[K_1 \;\; K_2] = \begin{bmatrix}-1-K_1 & -0.5-K_2\\ 1 & 0\end{bmatrix}$ $\qquad \alpha_c(z) = z^2$

$$|zI - \Phi + \Gamma K| = \begin{vmatrix} z+1+k_1 & 0.5+k_2 \\ -1 & z \end{vmatrix} = z^2 + (k_1+1)z + 0.5 + k_2$$

$k_1 = -1$

$k_2 = -0.5$

$K = [-1 \quad -0.5]$

b) $$\Phi - LH\Phi = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}\begin{bmatrix} -1.5 & -0.5 \end{bmatrix} = \begin{bmatrix} -1+1.5l_1 & -0.5+0.5l_1 \\ 1+1.5l_2 & 0.5l_2 \end{bmatrix}$$

$$|zI - \Phi + LH\Phi| = \begin{vmatrix} z-1.5l_1+1 & 0.5-0.5l_1 \\ -1-1.5l_2 & z-0.5l_2 \end{vmatrix} = z^2 + (1-1.5l_1-0.5l_2)z \; - 0.5 l_2 + \cancel{0.75l_1l_2} + 0.5 - 0.5l_1$$

$$\cancel{\quad} + 0.75l_2 - \cancel{0.75l_1l_2}$$

$$\alpha_e(z) = (z-0.1-0.1j)(z-0.1+0.1j) = z^2 - 0.2z + 0.02$$

$$z^2 + (1-1.5l_1-0.5l_2)z + 0.5 + 0.5l_1 + 0.25l_2$$

$-0.5l_1 + 0.25l_2 = -0.48$

$1.5l_1 + 0.5l_2 = 1.2$

$2.5l_1 = 2.16$

$l_1 = 0.8640 \rightarrow l_2 = 2.4 - 3l_1 = -0.192$

$$L = \begin{bmatrix} 0.864 \\ -0.192 \end{bmatrix}$$

c) Função de transferência do compensador:

Eqs. estado estimador dinâmico: $\hat{x}(k+1) = (\Phi - LH\Phi)\hat{x}(k) + (\Gamma - LH\Gamma)\underbrace{u(k)}_{-K\hat{x}(k)} + Ly(k+1)$

$$\hat{x}(k+1) = \underbrace{(\Phi - LH\Phi - \Gamma K + LH\Gamma K)}_{\Phi_D}\hat{x}(k) + Ly(k+1)$$

$$u(k) = -K\hat{x}(k)$$

$$\Phi_D = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0.864 \\ -0.192 \end{bmatrix}\begin{bmatrix} -1.5 & -0.5 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}[-1 \quad -0.5] + \begin{bmatrix} 0.864 \\ -0.192 \end{bmatrix}[1 \quad -0.5]\begin{bmatrix} 1 \\ 0 \end{bmatrix}[-1 \quad -0.5]$$

$$\Phi_D = \begin{bmatrix} 0.4320 & 0 \\ 0.9040 & 0 \end{bmatrix}$$

$$\mathcal{Z}\left|\begin{array}{l} \hat{x}(k+1) = \Phi_D\hat{x}(k) + Ly(k+1) \\ u(k) = -K\hat{x}(k) \end{array}\right. \rightarrow \quad zX(z) = \Phi_D X(z) + LzY(z)$$

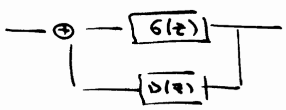
$$X(z) = (zI - \Phi_D)^{-1}LzY(z)$$

$$\frac{U(z)}{Y(z)} = -K(zI-\Phi_D)^{-1}Lz$$

$$\frac{U(z)}{Y(z)} = -[-1 \quad -0.5]\begin{bmatrix} z-0.4320 & 0 \\ -0.9040 & z \end{bmatrix}^{-1}\begin{bmatrix} 0.864 \\ -0.192 \end{bmatrix}z$$

$$= \frac{[1 \quad 0.5]\begin{bmatrix} z & 0 \\ 0.904 & z-0.4320 \end{bmatrix}\begin{bmatrix} 0.864 \\ -0.192 \end{bmatrix}\cancel{z}}{\cancel{z}(z-0.432)} = \frac{[z+0.452 \quad \frac{z}{2}-0.216]\begin{bmatrix} 0.864 \\ -0.192 \end{bmatrix}}{(z-0.432)} = \frac{0.768z+0.432}{z-0.432}$$

d) $$G(z) = \frac{z-0.5}{z^2+z+0.5} \leftarrow (z+0.5\pm0.5j)$$



se $\bar{N}=1$: $$\frac{Y(z)}{R(z)} = \frac{G(z)}{1-G(z)D(z)} = \frac{(z-0.5)(z-0.432)}{(z^2+z+0.5)(z-0.432)-(z-0.5)(0.768z+0.432)}$$

$$\frac{Y(z)}{R(z)} = \frac{z^2-0.932z+0.216}{z^3-0.2z^2+0.02z} \rightarrow \text{pólos: } 0,\; 0.1+0.1j,\; 0.1-0.1j$$

$$\lim_{z\to1}\frac{Y(z)}{R(z)} = 0.3463$$

$\bar{N} = 2.887$

(4) seja $z(k) = P\,x(k)$, $P = \begin{bmatrix} 1 & -0.5 \\ 0 & 1 \end{bmatrix}$, $x = Tz$ $\longrightarrow$ $\hat{x}_1 = \hat{z}_1 + 0.5\,\hat{z}_2$

$\hat{x}_2 = \hat{z}_2$

~~$\Phi_z = T^{-1}\Phi T$~~

$$\Phi_z = T^{-1}\Phi T = \begin{bmatrix} 1 & -0.5 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} = \begin{array}{cc} \Phi_{aa} & \Phi_{ab} \\ \left[\begin{array}{c|c} -1.5 & -1.25 \\ \hline 1 & 0.5 \end{array}\right] & \\ \Phi_{ba} & \Phi_{bb} \end{array}$$

$$\Gamma_z = T^{-1}\Gamma = \begin{array}{c} \begin{bmatrix} 1 \\ \hline 0 \end{bmatrix} \end{array} \begin{array}{c} \Gamma_a \\ \Gamma_b \end{array}$$

$$H_z = HT = [1 \quad -0.5]\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix} = [1 \quad 0]$$

FICOU FALTANDO NO ENUNCIADO; POLO DO ERRO DE ESTIMAÇÃO EM $z = 0.1$.

$$\alpha_e(z) = \cancel{z+0.1}\ \ z - 0.1$$

$$|zI - \Phi_{bb} + L\Phi_{ab}| = z - 0.5 - 1.25L = z - 0.1$$

$$-1.25L = 0.4 \longrightarrow L = -0.32$$

amostra $y(k)$

IMPLEMENTAÇÃO: $z_1 \overset{(k)}{\cancel{x_2}}\ y(k)$

$$\hat{z}_2(k) = \Phi_{bb}\hat{z}_2(k-1) + \Phi_{ba}\,\overset{z_1}{\cancel{x_1}}(k-1) + \Gamma_b u(k-1) + L\big(z_1(k) - \Phi_{aa}z_1(k-1) - \Gamma_a u(k-1) - \Phi_{ab}\,\cancel{x_2}\,\hat{z}_2(k-1)\big)$$

$$\hat{z}_2(k) = \underbrace{\underbrace{(\Phi_{bb} - L\Phi_{ab})}_{0.5+0.32\times(-1.25)=0.1}\hat{z}_2(k-1) + \underbrace{(\Phi_{ba} - L\Phi_{aa})}_{1+0.32\times(-1.5)=0.52}z_1(k-1) + \underbrace{(\Gamma_b - L\Gamma_a)}_{0.32}u(k-1)}_{\text{passado}} + \underbrace{\underbrace{L}_{-0.32}z_1(k)}_{\text{atual}}$$

$$\hat{x}_1(k) = \hat{z}_1(k) + \overset{0.5}{\cancel{\ }}\hat{z}_2(k)$$

$$\hat{x}_2(k) = \hat{z}_2(k)$$

ENTÃO, O PROGRAMA DE CONTROLE DIGITAL FICA:

(k-1) CONVERSOR A/D $\longrightarrow y(k-1)$

$(z_1(k-1) = y(k-1))$

$\hat{z}_2(k-1) = \check{z}_2(k-1)\ \cancel{+}\ - 0.32\,y(k-1)$

$\hat{x}_1(k-1) = \overset{y(k-1)}{\cancel{z_1(k)}} + 0.5\,\hat{z}_2(k-1)$

$\hat{x}_2(k-1) = \hat{z}_2(k-1)$

$u(k-1) = -k_1\hat{x}_1(k-1) - k_2\hat{x}_2(k-1)$

$u(k-1) \longrightarrow$ CONVERSOR D/A

$\check{z}_2(k) = 0.1\,\hat{z}_2(k-1) + 0.52\,\overset{y}{\cancel{z_1}}(k-1) + 0.32\,u(k-1)$

ESPERA

(k) CONVERSOR A/D $\longrightarrow y(k)$

$(z_1(k) = y(k))$

$\hat{z}_2(k) = \check{z}_2(k) - 0.32\,y(k)$

$\hat{x}_1(k) = y(k) + 0.5\,\hat{z}_2(k)$

$\hat{x}_2(k) = \hat{z}_2(k)$

$u(k) = -k_1\hat{x}_1(k) - k_2\hat{x}_2(k)$

$u(k) \longrightarrow$ CONVERSOR D/A

$\check{z}_2(k+1) = 0.1\,\hat{z}_2(k) + 0.52\,y(k) + 0.32\,u(k)$

ESPERA

... E ASSIM POR DIANTE.

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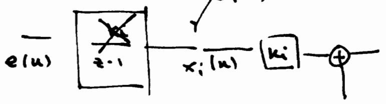
$x(k+1) = \Phi x(k) + \Gamma u(k)$ $\qquad y(k) = Hx(k)$

$x_i(k+1) = x_i(k) + \cancel{y(k)}$ $\qquad Hx(k) - r(k)$

$x_i(k+1) = x_i(k) + e(k)$



$x_i(k+1) = x_i(k) + \cancel{r(k) - Hx(k)}$ $\qquad Hx(k) - r(k)$

a)
$$\begin{bmatrix} x_i(k+1) \\ x(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & H \\ 0 & \Phi \end{bmatrix}}_{\Phi_i} \begin{bmatrix} x_i(k) \\ x(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \Gamma \end{bmatrix}}_{\Gamma_i} u(k) - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(k)$$

$$\Phi_i - \Gamma_i K = \begin{bmatrix} 1 & \cancel{1} & -0.5 \\ 0 & -1 & -0.5 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -0.5 \\ -k_1 & -1-k_2 & -0.5-k_3 \\ 0 & 1 & 0 \end{bmatrix}$$

$$|zI - \Phi_i + \Gamma_i K| = \begin{vmatrix} z-1 & -1 & 0.5 \\ k_1 & z+1+k_2 & 0.5+k_3 \\ 0 & -1 & z \end{vmatrix} = z^3 + (k_2)z^2 + (-1-k_2)z - 0.5k_1 + (0.5+k_3)z - 0.5 - k_3 + k_1 z$$

$$= z^3 + k_2 z^2 + (k_1 - k_2 + k_3 - 0.5)z - 0.5k_1 - k_3 - 0.5$$

$z^2 - 0.2z + 0.02$

$$\alpha_c(z) = (z-0.1)(z-0.1+0.1j)(z-0.1-0.1j) = z^3 - 0.3z^2 + 0.04z - 0.002$$

$z^3 - 0.2z^2 + 0.02z$

$-0.1z^2 + 0.02z - 0.002$

$k_2 = -0.3$

$k_1 + k_3 = 0.24$

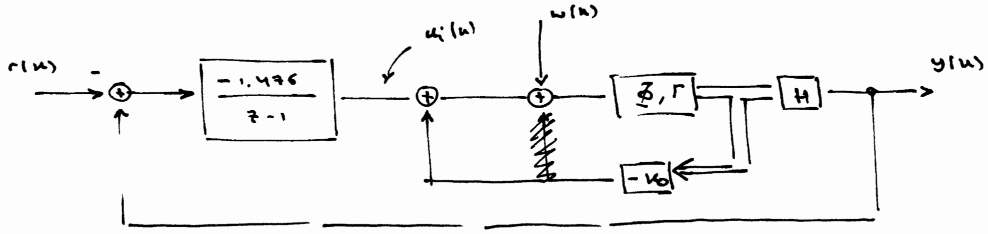
$-0.5k_1 - k_3 = 0.498$

$0.5k_1 = 0.738 \rightarrow k_1 = 1.476$

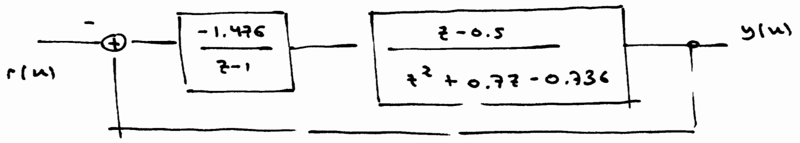
$k_3 = -1.236$

$$K = \left[\ \underbrace{1.476}_{K_i}\ \middle|\ \underbrace{-0.3 \quad -1.236}_{K_o}\ \right]$$

b)



$\frac{Y(z)}{R(z)}$:

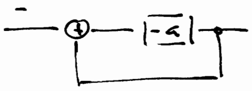


$$\begin{cases} x(k+1) = (\Phi - \Gamma K_o)x(k) + \Gamma \cancel{u_i(k)} \\ y(k) = Hx(k) \end{cases}$$

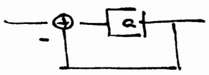
$$\Phi - \Gamma K_o = \begin{bmatrix} -0.7 & 0.736 \\ 1 & 0 \end{bmatrix}$$

$$G_c(z) = H(zI - \Phi + \Gamma K_o)^{-1}\Gamma = [1 \quad -0.5] \begin{bmatrix} z+0.7 & -0.736 \\ -1 & z \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{[1 \quad -0.5] \begin{bmatrix} z & 0.736 \\ 1 & z+0.7 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{z^2 + 0.7z - 0.736}$$

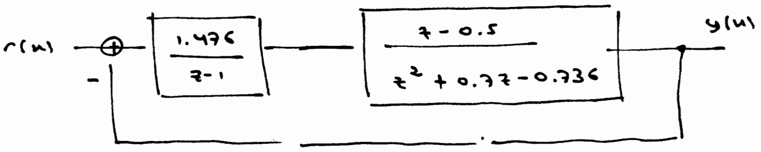
$$= \frac{z - 0.5}{z^2 + 0.7z - 0.736}$$



$$\frac{-a}{1+a} = \frac{a}{1+a}$$



$$= \frac{a}{1+a}$$



$$\frac{\dfrac{1.476(z-0.5)}{(z-1)(z^2+0.7z-0.736)}}{1+\dfrac{1.476(z-0.5)}{(z-1)(z^2+0.7z-0.736)}} = \frac{Y(z)}{R(z)} = \frac{1.476(z-0.5)}{z^3-0.3z^2+(-0.736\cancel{+}-0.7)z+0.736+1.476z-\dfrac{1.476}{2}}$$

$$= \frac{1.476(z-0.5)}{z^3-0.3z^2+0.04z\ \cancel{\#}-0.002}$$

NOTE QUE: $\lim\limits_{z\to 1}\dfrac{Y(z)}{R(z)} = 1.0$ //

$\dfrac{Y(z)}{W(z)}$:

$$\begin{bmatrix} x_i(k+1) \\ x(k+1)\end{bmatrix} = \begin{bmatrix} 1 & H \\ 0 & \Phi \end{bmatrix}\begin{bmatrix} x_i(k) \\ x(k)\end{bmatrix} + \begin{bmatrix} 0 \\ \Gamma \end{bmatrix}\overset{-K_i x_i - K_0 x}{u(k)} + \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} w(k) - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\cancel{r(k)}\ 0$$

$$\begin{bmatrix} x_i(k+1) \\ x(k+1)\end{bmatrix} = \underbrace{\begin{bmatrix} 1 & H \\ -\Gamma K_i & \Phi-\Gamma K_0 \end{bmatrix}}_{\Phi_w}\begin{bmatrix} x_i(k) \\ x(k)\end{bmatrix} + \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} w(k)$$

$$y(k) = [\,0 \quad H\,]\begin{bmatrix} x_i(k) \\ x(k)\end{bmatrix}$$

$$(zI-\Phi_w)^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$zI-\Phi_w = \begin{bmatrix} z-1 & -1 & 0.5 \\ 1.476 & z+0.7 & -0.736 \\ 0 & -1 & z \end{bmatrix}$$

$$\frac{Y(z)}{W(z)} = [\,0 \quad H\,](zI-\Phi_w)^{-1}\begin{bmatrix} 0 \\ \Gamma \end{bmatrix} = [\,0 \quad 1 \quad -0.5\,](zI-\Phi_w)^{-1}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = [\,0 \quad 1 \quad -0.5\,]\begin{bmatrix} b \\ e \\ h \end{bmatrix} = e-\frac{h}{2}$$

$$e = \frac{(z+0.7)\big((z-1)z\big)}{z^3-0.3z^2-\cancel{0.7z}_{0.7}-\dfrac{1.476}{2}-0.736z+0.736+1.476z} = \frac{z^3-0.3z^2-0.7z}{z^3-0.3z^2+0.04z-0.002}$$

$$\cancel{h = \frac{(-1)(-1)\left(-0.736z+0.736-\frac{1.476}{2}\right)}{z^3-0.3z^2+0.04z-0.002} = \frac{-0.736z-0.002}{z^3-0.3z^2+0.04z-0.002}}$$

$$\cancel{\frac{Y(z)}{W(z)} = \frac{z^3-0.3z^2+\left(-0.7+\frac{0.736}{2}\right)z+0.001}{z^3-0.3z^2+0.04z-0.002}}$$

$$h = \frac{(-1)(-0.736)(-1)(z-1)}{z^3-0.3z^2+0.04z-0.002} \quad \leadsto \quad \frac{Y(z)}{W(z)} = \frac{z^3-0.3z^2-0.7z+\dfrac{0.736z}{2}-\dfrac{0.736}{2}}{z^3-0.3z^2+0.04z-0.002}$$

$$\frac{Y(z)}{W(z)} = \frac{z^3-0.3z^2-0.332z-0.368}{z^3-0.3z^2+0.04z-0.002}$$

NOTE QUE $\lim\limits_{z\to 1}\dfrac{Y(z)}{W(z)} = 0$ //

OBS.: VARIAÇÃO INTERESSANTE DA QUESTÃO 3:

— NA QUESTÃO 3, USANDO UM ESTIMADOR DE PREDIÇÃO:

$$\Phi - LH = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \ell_2 \end{bmatrix} \begin{bmatrix} \cancel{-1.5}\ 1 & -0.5 \end{bmatrix} = \begin{bmatrix} -1-\ell_1 & -0.5+0.5\ell_1 \\ 1-\ell_2 & 0.5\ell_2 \end{bmatrix}$$

$$|zI - \Phi + LH| = \begin{vmatrix} z+1+\ell_1 & 0.5-0.5\ell_1 \\ -1+\ell_2 & z-0.5\ell_2 \end{vmatrix} = z^2 + (1+\ell_1-0.5\ell_2)z - 0.5\ell_2 - \cancel{0.5\ell_1\ell_2} + 0.5 - 0.5\ell_1 - 0.5\ell_2 + \cancel{0.5\ell_1\ell_2}$$

$$= z^2 + (1+\ell_1-0.5\ell_2)z + 0.5 - 0.5\ell_1 - \ell_2$$

$$\alpha_e(z) = z^2 - 0.2z + 0.02$$

$$\ell_1 - 0.5\ell_2 = -1.2$$
$$-0.5\ell_1 - \ell_2 = -0.48$$
$$-2.5\ell_2 = -0.24 \qquad \ell_2 = 0.096$$
$$\ell_1 = -1.2 + \frac{0.096}{2} = -1.152$$

$$\ell_1 - 0.5\ell_2 = -1.2$$
$$0.5\ell_1 + \ell_2 = 0.48$$
$$2.5\ell_1 = 0.48 - 2.4 \longrightarrow \ell_1 = -0.768$$
$$\ell_2 = 0.48 + \frac{0.768}{2} = 0.864 \qquad L = \begin{bmatrix} -0.768 \\ 0.864 \end{bmatrix}$$

FUNÇÃO DE TRANSFERÊNCIA DO COMPENSADOR:

EQS. ESTADO ESTIMADOR PREDIÇÃO: $\hat{x}(k+1) = (\Phi - \Gamma K - LH)\hat{x}(k) + Ly(k)$

$u(k) = -K\hat{x}(k)$

$$\Phi_0 = \Phi - \Gamma K - LH = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [-1 \quad -0.5] - \begin{bmatrix} -0.768 \\ 0.864 \end{bmatrix} [1 \quad -0.5] = \begin{bmatrix} 0.768 & -0.384 \\ 0.136 & 0.432 \end{bmatrix}$$

$$\frac{U(z)}{Y(z)} = -[-1 \quad -0.5] \underbrace{\begin{bmatrix} z-0.768 & 0.384 \\ -0.136 & z-0.432 \end{bmatrix}^{-1}}_{\frac{\begin{bmatrix} z-0.432 & -0.384 \\ 0.136 & z-0.768 \end{bmatrix}}{(z^2) + (-0.432-0.768)z \cancel{-}+ 0.136\times0.384 + 0.432\times0.768}} \begin{bmatrix} -0.768 \\ 0.864 \end{bmatrix} = \frac{\overset{-0.336}{\cancel{-0.3882}}z + \overset{-0.384}{\cancel{0.2995}}}{z^2 - 1.2z + \underset{0.384}{\cancel{0.3522}}}$$

$$\frac{G(z)}{1-G(z)D(z)} = \frac{(z-0.5)\cancel{(\ldots)}(z^2-1.2z+0.384)}{(z^2+z+0.5)(z^2-1.2z+0.384) - (z-0.5)(-0.336z-0.384)}$$

$$= \frac{z^3 - 1.7z^2 + 0.984z - 0.192}{z^4 - 0.2z^3 + 0.02z^2} \longrightarrow \text{PÓLOS: } 0, 0, 0.1 \pm 0.1j$$

$$\lim_{z\to1} \frac{Y(z)}{R(z)} = 0.1122 \quad \rightsquigarrow \quad \bar{N} = 8.913$$

—— " ——