Lista#10 Controle 2 : ——— Gabarito

① $G(s) = \dfrac{2/3}{s+1} + \dfrac{2/3}{s+2}$

a.1) $x' = \begin{pmatrix} -1 & \\ & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$

$y = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} \end{pmatrix} x$

$\Phi = e^{F T}$ $\qquad \Phi = \begin{bmatrix} e^{-T} & \\ & e^{-2T} \end{bmatrix}$ $\qquad T = 0.05 \;:\; \Phi = \begin{bmatrix} 0.9512 & \\ & 0.9048 \end{bmatrix}$

$\Gamma = \int_0^T e^{F\sigma} G\, d\sigma = \int_0^T \begin{bmatrix} e^{-\sigma} \\ e^{-2\sigma} \end{bmatrix} d\sigma = \left. \begin{matrix} -e^{-\sigma} \\ -\dfrac{e^{-2\sigma}}{2} \end{matrix} \right|_0^T = \begin{pmatrix} 1-e^{-T} \\ \dfrac{1-e^{-2T}}{2} \end{pmatrix}$ $\qquad T = 0.05 \;:\; \Gamma = \begin{bmatrix} 0.0488 \\ 0.0476 \end{bmatrix}$

$G(z) = H(zI - \Phi)^{-1}\Gamma$

$= \dfrac{2}{3} [1 \quad 1] \underbrace{\begin{bmatrix} z-e^{-T} & 0 \\ 0 & z-e^{-2T} \end{bmatrix}^{-1}}_{\frac{\begin{bmatrix} z-e^{-2T} & \\ & z-e^{-T} \end{bmatrix}}{(z-e^{-T})(z-e^{-2T})}} \begin{bmatrix} 1-e^{-T} \\ \dfrac{1-e^{-2T}}{2} \end{bmatrix}$

① $z^2 - (e^{-T} + e^{-2T}) z + e^{-3T}$

② $(z - e^{-2T})(1-e^{-T}) = (1-e^{-T}) z - (1-e^{-T}) e^{-2T}$

③ $(z-e^{-T}) \dfrac{(1-e^{-2T})}{2} = \dfrac{(1-e^{-2T})}{2} z - \dfrac{(1-e^{-2T})}{2} e^{-T}$

$+$

$\left(\dfrac{3}{2} - e^{-T} - \dfrac{e^{-2T}}{2}\right) z - \left(e^{-2T} - e^{-3T} + \dfrac{1}{2} e^{-T} - \dfrac{e^{-3T}}{2}\right)$

$\left(\dfrac{3}{2} - e^{-T} - \dfrac{e^{-2T}}{2}\right) z - \left(\dfrac{1}{2} e^{-T} + e^{-2T} - \dfrac{3}{2} e^{-3T}\right)$

$G(z) = \dfrac{\dfrac{2}{3}\left[\overbrace{\left(\dfrac{3}{2} - e^{-T} - \dfrac{e^{-2T}}{2}\right)}^{0.0964} z - \overbrace{\left(\dfrac{1}{2} e^{-T} + e^{-2T} - \dfrac{3}{2} e^{-3T}\right)}^{0.0894}\right]}{z^2 - \underbrace{(e^{-T} + e^{-2T})}_{1.8561} z + \underbrace{e^{-3T}}_{0.8607}} = \dfrac{0.0642\, z - 0.0596}{z^2 - 1.8561\, z + 0.8607}$

(Pólos: 0.9521 & 0.9040)

Alternativa:

$G(z) = \dfrac{2}{3} [1 \quad 1] \underbrace{\begin{bmatrix} z-0.9512 & \\ & z-0.9048 \end{bmatrix}^{-1}}_{\frac{\begin{bmatrix} z-0.9048 & \\ & z-0.9512 \end{bmatrix}}{z^2 - 1.8560z + 0.8606}} \begin{bmatrix} 0.0488 \\ 0.0476 \end{bmatrix} = \dfrac{0.0643\, z - 0.0596}{z^2 - 1.8560\, z + 0.8606}$

$\begin{pmatrix} 0.9522 \\ 0.9038 \end{pmatrix}$

c.2) EQUIV. DEGRAU :

$$\frac{G(s)}{s} = \frac{\frac{4}{3}s + 2}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} = \frac{1}{s} - \frac{\frac{2}{3}}{s+1} - \frac{\frac{1}{3}}{s+2}$$

$A = 1$

$$B = \frac{2 - \frac{4}{3}}{-1} = -\frac{2}{3}$$

$$C = \frac{2 - \frac{8}{3}}{2} = -\frac{1}{3}$$

$$\cancel{s^2} + 3s + 2 - \frac{2}{3}\cancel{s^2} - \frac{4}{3}s - \cancel{\frac{1}{3}s^2} - \frac{s}{3}$$

$$\frac{9-5}{3}s + 2 \quad \text{ok.}$$

$$\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right) = \left(1 - \frac{2}{3}e^{-t} - \frac{1}{3}e^{-2t}\right)u(t) = y(t)$$

$$y(k) = \left(1 - \frac{2}{3}e^{-Tk} - \frac{1}{3}e^{-2Tk}\right)u(k)$$

$$Y(z) = \frac{z}{z-1} - \frac{\frac{2}{3}z}{z - e^{-T}} - \frac{\frac{1}{3}z}{z - e^{-2T}}$$

$$\frac{z-1}{z}Y(z) = 1 - (z-1)\left(\frac{\frac{2}{3}}{z - e^{-T}} + \frac{\frac{1}{3}}{z - e^{-2T}}\right) = G(z)$$

$$\frac{z^2 - (e^{-T} + e^{-2T})z + e^{-3T} - (z-1)\left(\frac{2}{3}z - \frac{2}{3}e^{-2T} + \frac{1}{3}z - \frac{1}{3}e^{-T}\right)}{z^2 - (e^{-T} + e^{-2T})z + e^{-3T}}$$

(4)

(4): $$z^2 - (e^{-T} + e^{-2T})z + e^{-3T} - \left(\underbrace{\frac{2}{3}z^2 - \frac{2}{3}e^{-2T}z + \frac{1}{3}z^2 - \frac{1}{3}e^{-T}z - \frac{2}{3}z + \frac{2}{3}e^{-2T} - \frac{1}{3}z + \frac{1}{3}e^{-T}}_{z^2 + \left(-1 - \frac{1}{3}e^{-T} - \frac{2}{3}e^{-2T}\right)z + \frac{1}{3}e^{-T} + \frac{2}{3}e^{-2T}}\right)$$

$$\left(+1 - \frac{2}{3}e^{-T} \cancel{-} \frac{1}{3}e^{-2T}\right)z + \left[-\frac{1}{3}e^{-T} \cancel{-} \frac{2}{3}e^{-2T} + e^{-3T}\right]$$

$$\left(1 - \frac{2}{3}e^{-T} + \frac{1}{3}e^{-2T}\right)z + \left(-\frac{1}{3}e^{-T} - \frac{2}{3}e^{-2T} + e^{-3T}\right) = 0.0642z - 0.0596$$

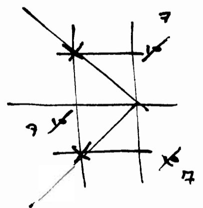
$$G(z) = \frac{0.0642z - 0.0596}{z^2 - 1.8561z + 0.8607}$$

b) FCC:

$$\begin{cases} x(k+1) = \begin{bmatrix} 1.8561 & -0.8607 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \\ y(k) = [\,0.0642 \quad -0.0596\,]\, x(k) \end{cases}$$

c) $t_r < 0.18$ seg $\quad \omega_n > \frac{1.8}{0.18} = 10$ rad/seg

$M_p < 5\%$ ⟶ $\xi > 0.7 \quad \theta > 45°$



~~T = 0.05~~

~~$s = -10 \pm 10j$ ⟶ $z = e^{sT} = e^{-0.5 \pm 0.5j}$~~

~~$e^{-0.5}(\cos 0.5 \pm j\sin 0.5)$~~

~~$z = 0.5323 \pm 0.2908j$~~

$$\alpha_c(z) = (z - 0.5323 + 0.2908j)(z - 0.5323 - 0.2908j)$$
$$= z^2 - 1.0646z + 0.3679$$

$$\begin{bmatrix} 1.8561 & -0.8607 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [k_1 \ k_2]$$

$$|zI - \Phi + \Gamma K| = \begin{vmatrix} z - 1.8561 + k_1 & 0.8607 + k_2 \\ -1 & z \end{vmatrix}$$

$$-1.8561 + k_1$$

$$|zI - \Phi| = 0 \quad \begin{vmatrix} z-1 & -T \\ 0 & z-1 \end{vmatrix} \quad z^2 - 2z + 1$$

$$s = -7 \pm 7j \longrightarrow z = e^{sT} = e^{-0.35 \pm 0.35j} \qquad (T = 0.05)$$

$$= e^{-0.35}(\cos 0.35 \pm j \sin 0.35) = 0.662 \pm 0.2416j$$

$$\alpha_c(z) = (z - 0.662 + 0.2416j)(z - 0.662 - 0.2416j) = z^2 - 1.324z + 0.4966$$

$$\begin{bmatrix} 1.8561 & -0.8607 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [k_1 \ k_2] = \begin{bmatrix} 1.8561 - k_1 & -0.8607 - k_2 \\ 1 & 0 \end{bmatrix}$$

$$|zI - \Phi + \Gamma K| = \begin{vmatrix} z - 1.8561 + k_1 & 0.8607 + k_2 \\ -1 & z \end{vmatrix} = z^2 + (-1.8561 + k_1)z + 0.8607 + k_2$$

$$-1.8561 + k_1 = -1.324 \longrightarrow k_1 = 0.5321$$

$$0.8607 + k_2 = 0.4966 \longrightarrow k_2 = -0.3641$$

$$K = [0.5321 \quad -0.3641]$$

d) $$\Phi - \Gamma K = \begin{bmatrix} 1.324 & -0.4966 \\ 1 & 0 \end{bmatrix}$$

$u(k) = -Kx(k) + \tilde{N}r$, assumindo $\tilde{N} = 1$ a princípio:

$$\begin{cases} x(k+1) = (\Phi - \Gamma K)x(k) + \Gamma r(k) \\ y(k) = Hx(k) \end{cases}$$

$$\frac{Y(z)}{R(z)} = H(zI - \Phi + \Gamma K)^{-1}\Gamma = [0.0642 \quad -0.0596] \frac{\begin{bmatrix} z & -0.4966 \\ 1 & z - 1.324 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{z^2 - 1.324z + 0.4966} = \frac{0.0642z - 0.0596}{z^2 - 1.324z + 0.4966}$$

$$\lim_{z \to 1} \frac{Y(z)}{R(z)} = 0.0267 \longrightarrow \tilde{N} = 37.45$$

Alternativo:

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} \Phi - I & \Gamma \\ H & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{array}{c|c} \begin{bmatrix} 217.39 \\ 217.39 \\ \hline 1 \end{bmatrix} & \begin{array}{c} N_x \\ \\ N_u \end{array} \end{array}$$

$$\tilde{N} = N_u + KN_x = 1 + [0.5321 \quad -0.3641] \begin{bmatrix} 217.39 \\ 217.39 \end{bmatrix} = 37.52$$

PROBLEMA #1 (MATLAB)

```
% ITEM (1d)

>> num=[4/3 2]; den=[1 3 2]; sys=tf(num,den);
>> a=0; b=15; pts=10000; stp=(b-a)/pts; t=a:stp:(b-
stp);
>> u=stepfun(t,5);
>> ya=lsim(sys,u,t); plot(t,ya,'k-'); hold on; % Linha
preta: resposta de G(s) ao degrau
>> axis([0 15 0 1.1]); grid on;
>> sysd=c2d(sys,0.05,'zoh')

Transfer function:
 0.06423 z - 0.05959
----------------------
z^2 - 1.856 z + 0.8607

Sampling time: 0.05
>> a=0; b=15; stp=0.05; t=a:stp:(b-stp);
u=stepfun(t,5);
>> yd=lsim(sysd,u,t); plot(t,yd,'k.'); % Pontos
pretos: resposta de G(z) ao degrau

% ITEM (1e)

>> Phi=[1.856 -0.8607 ; 1 0]; Gamma=[1 ; 0];
H=[0.06423 -0.05959];
>> K=place(Phi,Gamma,[exp(-0.35-0.35*j) exp(-
0.35+0.35*j)])

K =
    0.5321   -0.3641

>> [n,d]=ss2tf(Phi-Gamma*K,Gamma,H,0)

n =
         0    0.0642   -0.0596

d =
    1.0000   -1.3239    0.4966

>> Nbar=sum(d)/sum(n)

Nbar =

   37.2103

>> sysdc=ss(Phi-Gamma*K,Gamma*Nbar,H,0,0.05)

a =
            x1       x2
   x1    1.324  -0.4966
   x2        1        0

b =
          u1
   x1  37.21
   x2      0

c =
             x1        x2
   y1   0.06423  -0.05959

d =
       u1
   y1   0

Sampling time: 0.05
Discrete-time model.
>> ydc=lsim(sysdc,u,t); plot(t,ydc,'k*'); % Asteriscos
pretos: resposta ao degrau (malha fechada)
>> axis([4 11 0 3.5]); grid on;

% Comentario: note o forte overshoot na resposta ao
degrau - o metodo dos polos dominantes
%             nao funciona bem, porque ha' um zero em
frequencia muito baixa:
%             >> roots(n) -> ans = 0.9278
```

PROBLEMA #3 (MATLAB)

```
% ITEM (3d)

>> num=26; den=[1 2 26]; sys=tf(num,den);
>> a=0; b=15; pts=10000; stp=(b-a)/pts; t=a:stp:(b-
stp);
>> u=stepfun(t,5);
>> ya=lsim(sys,u,t); plot(t,ya,'k-'); hold on; % Linha
preta: resposta de G(s) ao degrau
>> sysd=c2d(sys,0.05,'zoh')

Transfer function:
 0.03127 z + 0.03025
----------------------
z^2 - 1.843 z + 0.9048

Sampling time: 0.05
>> a=0; b=15; stp=0.05; t=a:stp:(b-stp);
u=stepfun(t,5);
>> yd=lsim(sysd,u,t); plot(t,yd,'k.'); % Pontos
pretos: resposta de G(z) ao degrau

% ITEM (3e)

>> Phi=[1.843 -0.9048 ; 1 0]; Gamma=[1 ; 0];
H=[0.03127 0.03025];
>> K=place(Phi,Gamma,[exp(-0.35-0.35*j) exp(-
0.35+0.35*j)])

K =

    0.5191   -0.4082

>> [n,d]=ss2tf(Phi-Gamma*K,Gamma,H,0)

n =
         0    0.0313    0.0303

d =
    1.0000   -1.3239    0.4966

>> Nbar=sum(d)/sum(n)

Nbar =

    2.8065

>> sysdc=ss(Phi-Gamma*K,Gamma*Nbar,H,0,0.05)

a =
            x1       x2
   x1    1.324  -0.4966
   x2        1        0

b =
          u1
   x1  2.806
   x2      0

c =
            x1       x2
   y1  0.03127  0.03025

d =
       u1
   y1   0

Sampling time: 0.05
Discrete-time model.
>> ydc=lsim(sysdc,u,t); plot(t,ydc,'k*'); % Asteriscos
pretos: resposta ao degrau (malha fechada)
>> axis([4 11 0 1.6]); grid on;

% Comentario: a resposta ao grau e' bastante proxima
%             da desejada. O overshoot e' aprox. 5%,
%             mas o tempo de subida e' ligeiramente
%             maior do que 0.18seg. Seria necessario
%             aumentar um pouco o modulo dos polos em
%             malha fechada.
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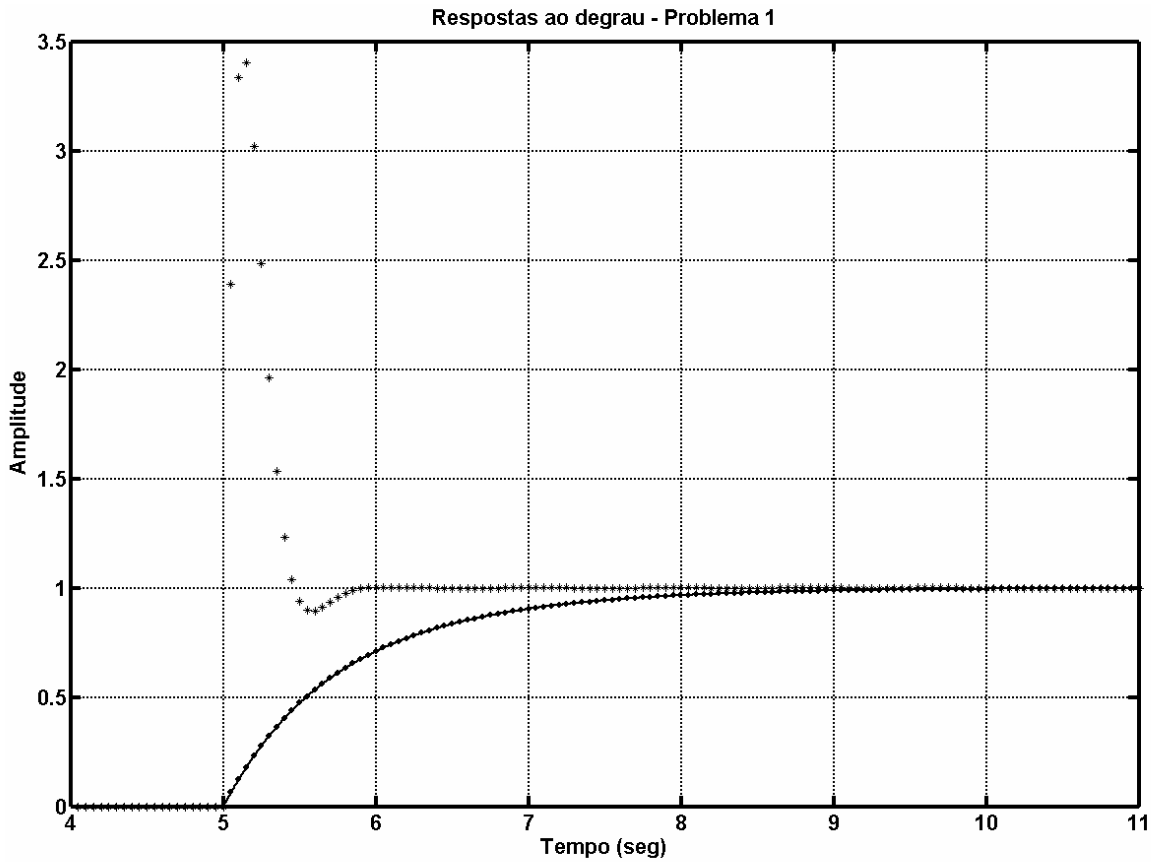

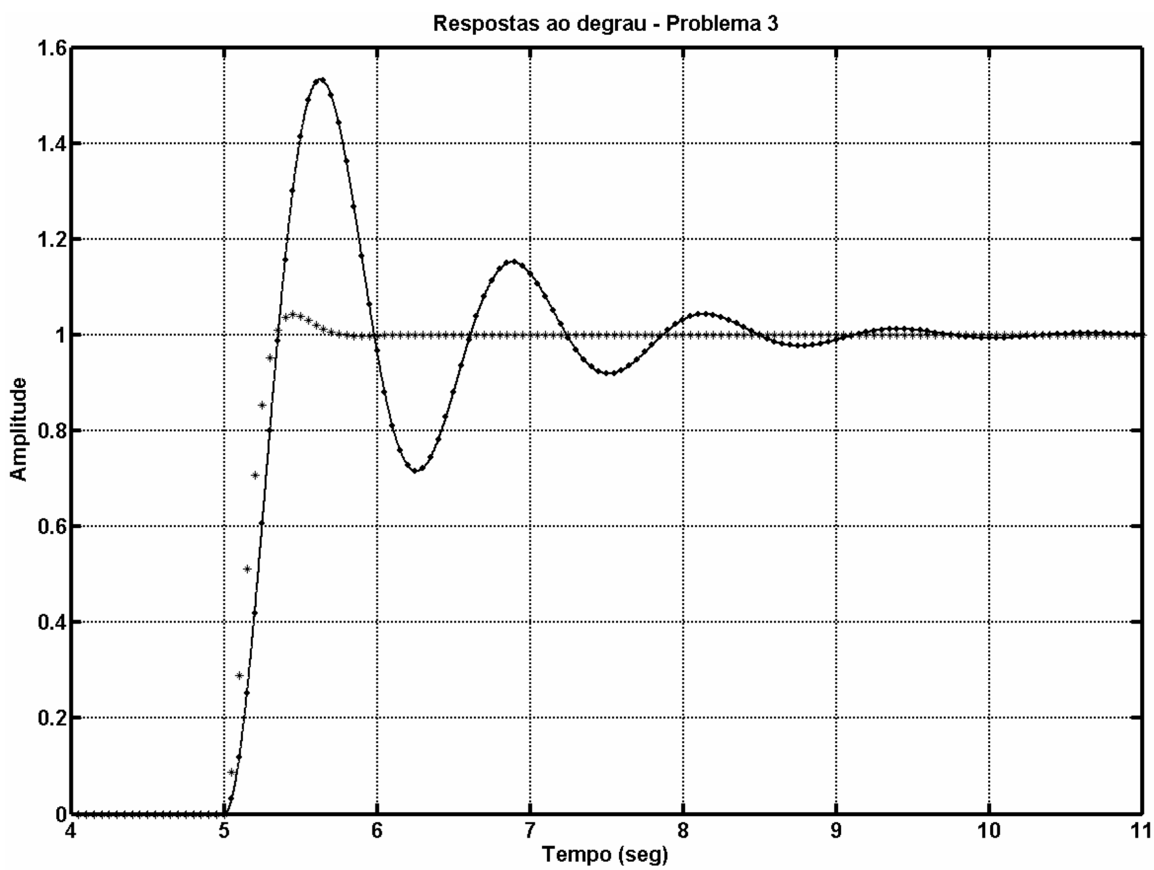

Pontos pretos: G(s)
Linha continua: G(z)
Asteriscos pretos: sistema discreto, em malha fechada

(2) $\mathcal{C} = [\Gamma \quad \Phi\Gamma] = \begin{bmatrix} \frac{T^2}{2} & \frac{3T^2}{2} \\ T & T \end{bmatrix}$

a) $\mathcal{C}^{-1} = \begin{bmatrix} -\frac{1}{T^2} & \frac{3}{2T} \\ \frac{1}{T^2} & -\frac{1}{2T} \end{bmatrix}$

$P = \begin{bmatrix} - p_1 - \\ - p_2 - \end{bmatrix} \qquad p_2 = [0 \quad 1]\,\mathcal{C}^{-1} = \begin{bmatrix} \frac{1}{T^2} & \frac{-1}{2T} \end{bmatrix}$

$p_1 = p_2 \Phi = \begin{bmatrix} \frac{1}{T^2} & \frac{-1}{2T} \end{bmatrix} \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{T^2} & \frac{1}{2T} \end{bmatrix}$

$P = \begin{bmatrix} \frac{1}{T^2} & \frac{1}{2T} \\ \frac{1}{T^2} & \frac{-1}{2T} \end{bmatrix} = T^{-1}$

$T = \begin{bmatrix} \frac{T^2}{2} & \frac{T^2}{2} \\ T & -T \end{bmatrix}$

b) $\Phi_c = T^{-1}\Phi T = \underbrace{\begin{bmatrix} \frac{1}{T^2} & \frac{1}{2T} \\ \frac{1}{T^2} & \frac{-1}{2T} \end{bmatrix} \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}}_{\begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \\ \frac{1}{T^2} & \frac{1}{2T} \end{bmatrix}} \begin{bmatrix} \frac{T^2}{2} & \frac{T^2}{2} \\ T & -T \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

$\Gamma_c = T^{-1}\Gamma = \begin{bmatrix} \frac{1}{T^2} & \frac{1}{2T} \\ \frac{1}{T^2} & \frac{-1}{2T} \end{bmatrix} \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Phi_c - \Gamma_c K_c = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [k_1 \quad k_2] = \begin{bmatrix} 2-k_1 & -1-k_2 \\ 1 & 0 \end{bmatrix}$

$|zI - \Phi_c + \Gamma_c K_c| = \begin{vmatrix} z-2+k_1 & 1+k_2 \\ -1 & z \end{vmatrix} = z^2 + (-2+k_1)z + 1 + k_2$

$\alpha_c(z) = z^2 - 1.6z + 0.7 \qquad -2+k_1 = -1.6 \longrightarrow k_1 = 0.4$

$1 + k_2 = 0.7 \longrightarrow k_2 = \cancel{0.3}\ -0.3$

$K_c = [0.4 \quad 0.3]$

c) $u = -K_c z = -\underbrace{K_c P}_{K} x \longrightarrow K = K_c P = [0.4 \quad -0.3] \begin{bmatrix} \frac{1}{T^2} & \frac{1}{2T} \\ \frac{1}{T^2} & \frac{-1}{2T} \end{bmatrix} = \begin{bmatrix} \frac{0.1}{T^2} & \frac{0.7}{2T} \end{bmatrix}$

$K = \begin{bmatrix} \frac{0.1}{T^2} & \frac{0.7}{2T} \end{bmatrix}$