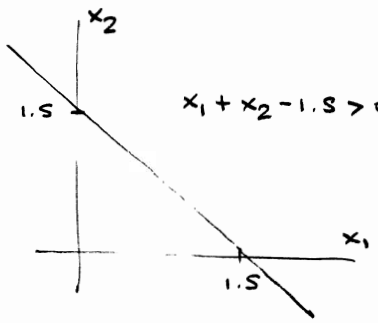
070717 GABARITO LISTA #2

QUESTÃO #1

a) REGIÕES DE DECISÃO

$x_1 + x_2 - 1.5 = 0$ (NEURÔNIO #1)



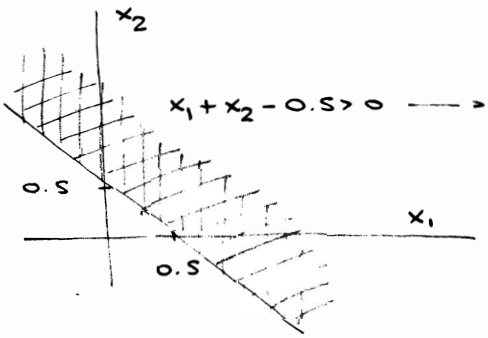
$x_1 + x_2 - 1.5 > 0 \longrightarrow o_1 = +1$

$x_1 + x_2 - 1.5 < 0$

$\hookrightarrow o_1 = 0$

SE $o_1 = 0$:

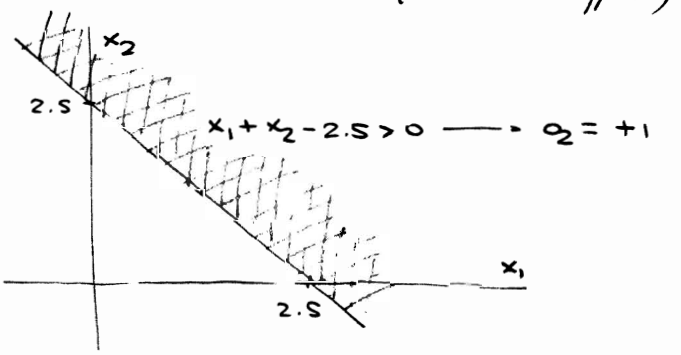
$x_1 + x_2 - 0.5 = 0$ (NEURÔNIO #2)



$x_1 + x_2 - 0.5 < 0 \longrightarrow o_2 = 0$

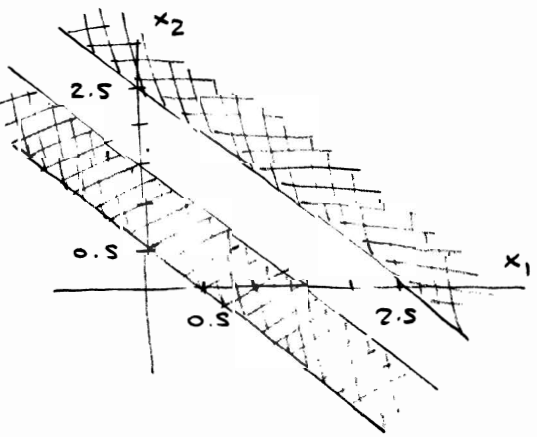
SE $o_1 = +1$:

$x_1 + x_2 - 2.5 = 0$ (NEURÔNIO #2)



$x_1 + x_2 - 2.5 < 0 \longrightarrow o_2 = 0$

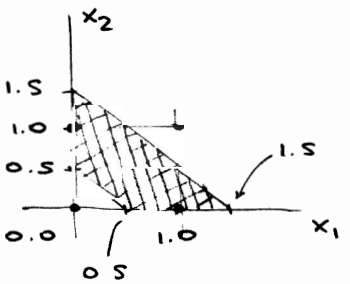
SAÍDA DA REDE NEURAL:



$o_2 = +1$

$o_2 = 0$

b) TABELA VERDADE



x_1	x_2	o_2
0	0	0
0	1	1
1	0	1
1	1	0

QUESTÃO #2:

$J(w) = k_1 (w - w_0)^2 + k_2$

$w = w_0 \longrightarrow J(w) = k_2 = J_{min}$; $\frac{dJ}{dw} = 2k_1 (w - w_0)$

ASSUMINDO $k_1 = 1$, $k_2 = 1$ E $w_0 = 1$

$w = 1 \longrightarrow J_{min} = 1$; $\frac{dJ}{dw} = 2(w-1)$

CHUTE INICIAL $w = 2.5$

TAXA DE APRENDIZADO $\eta = 0.10$; DECAIMENTO $\alpha = 1$ (η FIXO)

CRITÉRIO DE PARADA: $\left| \frac{\Delta J}{J} \right| < 0.01$ (1%)

① BACKPROPAGATION SEM MOMENTO ($\mu = 0$):

ITERAÇÃO $i = 1$

$w = 2.5$

$\frac{dJ}{dw} = 3$

$\Delta w = -0.1 \times 3 = -0.3$

$w = 2.5 - 0.3 = 2.2$

$J(2.2) = 2.44$

ITERAÇÃO $i = 2$

$w = 2.2$

$\frac{dJ}{dw} = 2.4$

$\Delta w = -0.1 \times 2.4 = -0.24$

$w = 2.2 - 0.24 = 1.96$

$J(1.96) = 1.92$

$\left| \frac{\Delta J}{J} \right| = \frac{|1.92 - 2.44|}{1.92} = 0.27$

ITERAÇÃO $i = 3$

$w = 1.96$

$\frac{dJ}{dw} = 1.92$

$\Delta w = -0.1 \times 1.92 = -0.19$

$w = 1.96 - 0.19 = 1.77$

$J(1.77) = 1.59$

$\left| \frac{\Delta J}{J} \right| = \frac{|1.59 - 1.92|}{1.59} = 0.21$

ITERAÇÃO $i = 4$

$w = 1.77$

$\frac{dJ}{dw} = 1.54$

$\Delta w = -0.1 \times 1.54 = -0.15$

$w = 1.77 - 0.15 = 1.62$

$J(1.62) = 1.38$

$\left| \frac{\Delta J}{J} \right| = \frac{|1.38 - 1.59|}{1.59} = 0.15$

ITERAÇÃO $i = 5$

$w = 1.62$

$\frac{dJ}{dw} = 1.24$

$\Delta w = -0.1 \times 1.24 = -0.12$

$w = 1.62 - 0.12 = 1.50$

$J(1.50) = 1.25$

$$\left|\frac{\Delta J}{J}\right| = \left|\frac{1.25 - 1.38}{1.25}\right| = 0.10$$

ITERAÇÃO $i = 6$

$w = 1.50$

$\frac{dJ}{dw} = 1.00$

$\Delta w = -0.1 \times 1.00 = -0.1$

$w = 1.50 - 0.1 = 1.40$

$J(1.40) = 1.16$

$$\left|\frac{\Delta J}{J}\right| = \left|\frac{1.16 - 1.25}{1.16}\right| = 0.08$$

ITERAÇÃO $i = 7$

$w = 1.40$

$\frac{dJ}{dw} = 0.80$

$\Delta w = -0.1 \times 0.8 = -0.08$

$w = 1.40 - 0.08 = 1.32$

$J(1.32) = 1.10$

$$\left|\frac{\Delta J}{J}\right| = \left|\frac{1.10 - 1.16}{1.10}\right| = 0.05$$

ITERAÇÃO $i = 8$

$w = 1.32$

$\frac{dJ}{dw} = 0.64$

$\Delta w = -0.1 \times 0.64 = -0.06$

$w = 1.32 - 0.06 = 1.26$

$J(1.26) = 1.07$

$$\left|\frac{\Delta J}{J}\right| = \left|\frac{1.07 - 1.10}{1.07}\right| = 0.03$$

ITERAÇÃO $i = 9$

$w = 1.26$

$\frac{dJ}{dw} = 0.52$

$\Delta w = -0.1 \times 0.52 = -0.05$

$w = 1.26 - 0.05 = 1.21$

$J(1.21) = 1.04$

$$\left|\frac{\Delta J}{J}\right| = \left|\frac{1.04 - 1.07}{1.04}\right| = 0.03$$

ITERAÇÃO $i = 10$

$w = 1.21$

$\frac{dJ}{dw} = 0.42$

$\Delta w = -0.1 \times 0.42 = -0.04$

$w = 1.17$

$J(1.17) = 1.03$

$$\left|\frac{\Delta J}{J}\right| = \left|\frac{1.03 - 1.04}{1.03}\right| = 0.0097 < \underline{0.01}$$

FIM. ($w = 1.21$ EM 10 ITERAÇÕES)

② BACKPROPAGATION COM MOMENTO ($\mu = 0.5$):

ITERAÇÃO $i = 1$

$w = 2.5$

$\frac{dJ}{dw} = 3$

$\Delta w = -0.1 \times 3 = -0.3$

$w = 2.5 - 0.3 = 2.2$

$J(2.2) = 2.44$

ITERAÇÃO $i = 2$

$w = 2.2$

$\frac{dJ}{dw} = 2.4$

$\Delta w = -0.1 \times 2.4 + 0.5 \times (-0.3) = -0.39$

$w = 2.2 - 0.39 = 1.81$

$J(1.81) = 1.66$; $\left|\frac{\Delta J}{J}\right| = \left|\frac{1.66 - 2.44}{1.66}\right|$ ($= 0.47$)

ITERAÇÃO $i = 3$

$w = 1.81$

$\frac{dJ}{dw} = 1.62$

$\Delta w = -0.1 \times 1.62 + 0.5 \times (-0.39) = -0.36$

$w = 1.81 - 0.36 = 1.45$

$J(1.45) = 1.20$

$$\left|\frac{\Delta J}{J}\right| = \left|\frac{1.20 - 1.66}{1.20}\right| = 0.38$$

ITERAÇÃO $i=4$

$w = 1.45$

$\frac{dJ}{dw} = 0.9$

$\Delta w = -0.1 \times 0.9 + 0.5 \times (-0.36) = -0.27$

$w = 1.45 - 0.27 = 1.18$

$J(1.18) = 1.03$

$\left| \frac{\Delta J}{J} \right| = \frac{|1.03 - 1.20|}{1.03} = 0.17$

ITERAÇÃO $i=5$

$w = 1.18$

$\frac{dJ}{dw} = 0.36$

$\Delta w = -0.1 \times 0.36 + 0.5 \times (-0.27) = -0.17$

$w = 1.18 - 0.17 = 1.01$

$J(1.01) = 1.00$

$\left| \frac{\Delta J}{J} \right| = \frac{|1.00 - 1.03|}{1.00} = 0.03$

ITERAÇÃO $i=6$

$w = 1.01$

$\frac{dJ}{dw} = 0.02$

$\Delta w = -0.1 \times 0.02 + 0.5 \times (-0.17) = -0.09$

$w = 1.01 - 0.09 = 0.92$

$J(0.92) = 1.01$

$\left| \frac{\Delta J}{J} \right| = \frac{|1.01 - 1.00|}{1.01} = 0.0099 < \underline{0.01}$

FIM. ($w = 0.92$ EM 6 ITERAÇÕES).

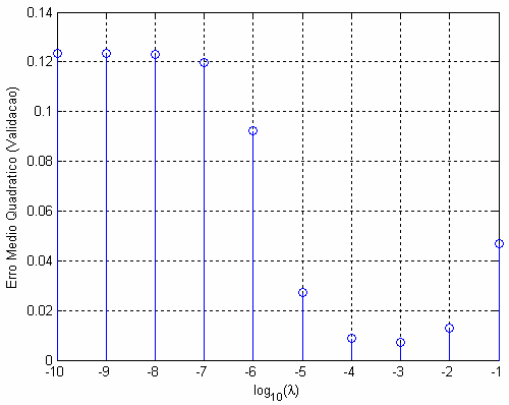
```
clear all;
xR = [0 0.2 0.4 0.6 0.8]'; tR = [-1.04 -0.39 0.09 0.92 1.43]';
XR = [ones(size(xR)) xR xR.^2 xR.^3 xR.^4];
xV = [0.1 0.3 0.5 0.7 0.9]'; tV = [-0.87 -0.07 0.62 1.10 1.72]';
XV = [ones(size(xV)) xV xV.^2 xV.^3 xV.^4];

% Item (a) - lambda de 1e-10 ate' 0.1:

lambda = [1e-10 1e-9 1e-8 1e-7 1e-6 1e-5 1e-4 1e-3 1e-2 1e-1];
E = zeros(length(lambda),2);
for k=1:length(lambda),
    w = inv(XR'*XR + lambda(k)*eye(5))*XR'*tR;
    E(k,:) = [mean((XR*w-tR).^2) mean((XV*w-tV).^2)];
end;
[log10(lambda)' E]

% ans =
%
%   -10.0000    0.0000    0.1235
%    -9.0000    0.0000    0.1234
%    -8.0000    0.0000    0.1231
%    -7.0000    0.0000    0.1196
%    -6.0000    0.0001    0.0923
%    -5.0000    0.0017    0.0272
%    -4.0000    0.0034    0.0089
%    -3.0000    0.0038    0.0071 <- Menor erro de validacao
%    -2.0000    0.0058    0.0128
%    -1.0000    0.0348    0.0467

stem(log10(lambda),E(:,2)); grid on;
xlabel('log_{10}(\lambda)'); ylabel('Erro Medio Quadratico
(Validacao)');
```

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% Item (b) - lambda = 0:

lambda = 0;
w = inv(XR'*XR + lambda*eye(5))*XR'*tR;
[mean((XR*w-tR).^2) mean((XV*w-tV).^2)]

% ans =
%
%     0.0000    0.1235

w

% w =
%
%    -1.0400
%     6.0292
%   -22.2604
%    48.0208
%   -30.9896

lambda = 1e-3;
w = inv(XR'*XR + lambda*eye(5))*XR'*tR;
[mean((XR*w-tR).^2) mean((XV*w-tV).^2)]

% ans =
%
%     0.0038    0.0071

w

% w =
%
%    -1.0184
%     2.6232
%     1.0544
%     0.1451
%    -0.9262

% Item (c) - Comparacoes

a = 0; b = 1; pts = 1000; stp = (b-a)/pts; x = (a:stp:(b-stp))';
X = [ones(size(x)) x x.^2 x.^3 x.^4];
lambda = 0;
w = inv(XR'*XR + lambda*eye(5))*XR'*tR;
figure; plot(xR,tR,'kx',xV,tV,'k+');
hold on; plot(x,X*w,'b-');
lambda = 1e-3;
w = inv(XR'*XR + lambda*eye(5))*XR'*tR;
plot(x,X*w,'r-'); grid on;
xlabel('x_{1}'); ylabel('x_{2}');
```