Quantizador Vetorial
Conceitos Básicos

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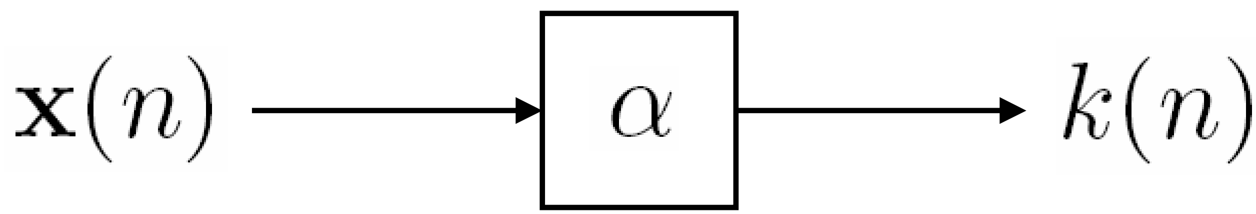
CPE718 – Aula #8 – Parte I

1. Codificador

- Dados de entrada (fonte): $\mathbf{X} \in \mathbb{R}^{M \times N}$

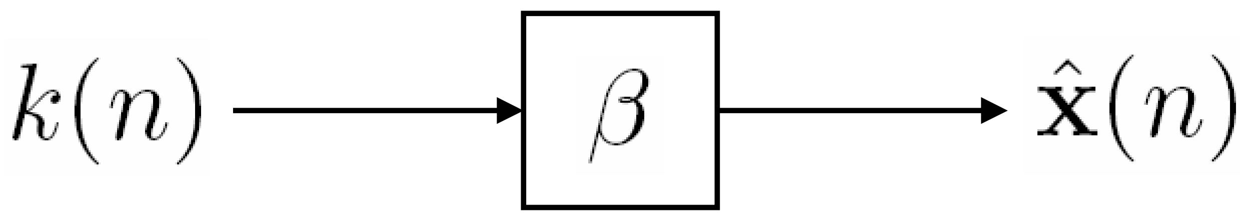
$$\mathbf{x}(n) \in \mathbb{R}^{M},\ n = 1, \ldots, N$$



2. Decodificador

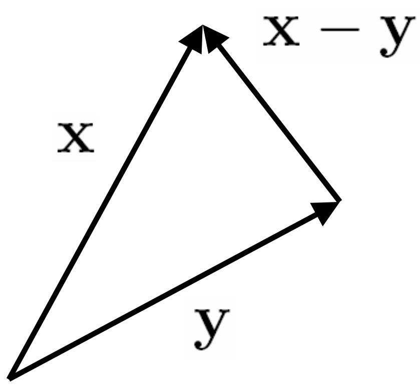
- Dicionário: $\mathbf{Y} \in \mathbb{R}^{M \times K}$

$k(n)$	1	2	3	$\cdots$	K
$\hat{\mathbf{x}}(n)$	$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\cdots$	$\mathbf{y}_K$



3. VQ

$$\hat{\mathbf{x}}(n) = \beta(\alpha(\mathbf{x}(n)))$$



$$d(\mathbf{x},\mathbf{y}) = ||\mathbf{x}-\mathbf{y}||^2 = \sum_{m=1}^{M}(x_m - y_m)^2$$

$$D = \frac{1}{N}\sum_{n=1}^{N} d(\mathbf{x}(n),\hat{\mathbf{x}}(n)) = \frac{1}{N}\sum_{n=1}^{N} ||\mathbf{x}(n)-\hat{\mathbf{x}}(n)||^2$$

$$D = E[d(\mathbf{x},\hat{\mathbf{x}})]$$

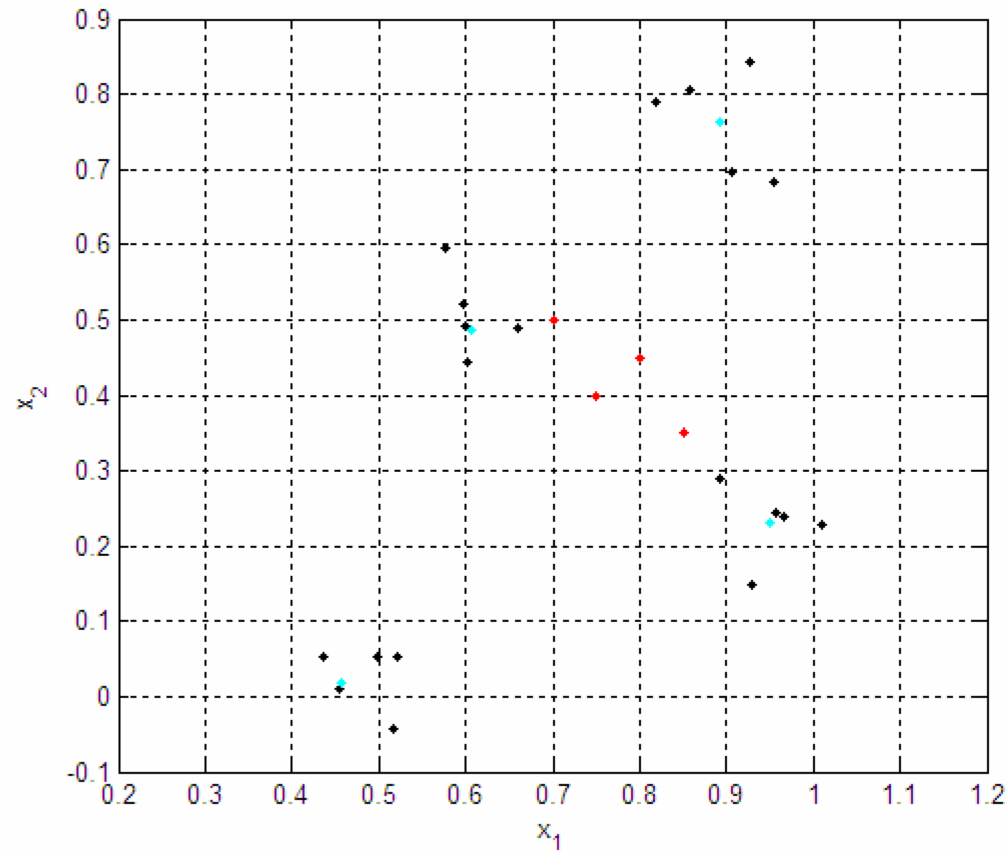
4. Problema (Projeto do VQ)

- Dado **X**, encontrar **Y** tal que $D = D_{\text{min}}$
- Exemplo (MATLAB):

$M = 2$

$N = 20$

$K = 4$



4. Problema (Código MATLAB)

```
>> close all; clear all;

>> randn('state',0); rand('state',0); M = 2; N = 20; K = 4; G = 4; e = 0.05;

>> C = rand(M,G); X = []; L = N/G;

>> for g = 1:G, X = [X repmat(C(:,g),1,L)+e*randn(M,L)]; end;

>> plot(X(1,:),X(2,:),'k.'); grid on;

>> hold on; plot(C(1,:),C(2,:),'c.');

>> Y = [ 0.7 0.75 0.8 0.85 ; 0.5 0.4 0.45 0.35];

>> plot(Y(1,:),Y(2,:),'r.');

>> xlabel('x_{1}'); ylabel('x_{2}');

>> axis([0.2 1.2 -0.1 0.9])
```

5. Considerações Básicas

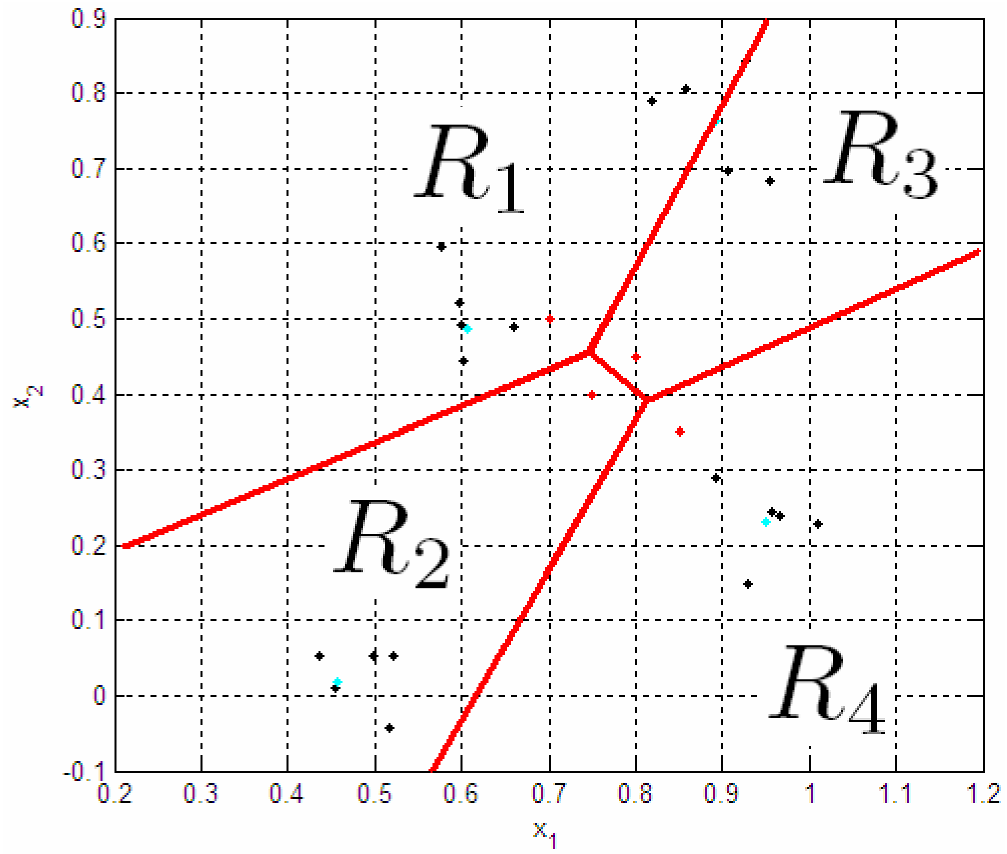
5.1. Condição da Partição (Codificador)

Fixando **Y**, calcular divisão de **X** em *K* sub-conjuntos

5.2. Condição do Centróide (Decodificador)

Fixando divisão de **X**, calcular **Y**

5.1. Condição da Partição




```
>> voronoi(Y(1,:),Y(2,:),'r-');
```

5.1. Condição da Partição

- Considerando que $\mathbf{y}_1$, $\mathbf{y}_2$, ..., $\mathbf{y}_K$ são conhecidos (dados):

$$D = \int d(\mathbf{x}, \hat{\mathbf{x}}) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

$$D \geq \int [\min_{k \in \{1,2,\ldots,K\}} d(\mathbf{x}, \mathbf{y}_k)] \cdot f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

- Particao otima:

$$R_i = \{\mathbf{x} \mid d(\mathbf{x}, \mathbf{y}_i) < d(\mathbf{x}, \mathbf{y}_j) \ \forall j \neq i\}$$

- Neste caso: $d(\mathbf{x}, \hat{\mathbf{x}}) = \min_{k \in \{1,2,\ldots,K\}} d(\mathbf{x}, \mathbf{y}_k)$

5.1. Condição da Partição

$$D = \frac{1}{N} \sum_{n=1}^{N} ||\mathbf{x}(n) - \hat{\mathbf{x}}(n)||^2$$

$$R_i = \{\mathbf{x} \mid d(\mathbf{x}, \mathbf{y}_i) < d(\mathbf{x}, \mathbf{y}_j) \ \forall j \neq i\}$$

$$D_{min} = \frac{1}{N} \sum_{n=1}^{N} ||\mathbf{x}(n) - \mathbf{y}_{k(n)}||^2$$

5.1. Condição da Partição

```
>> X

X =

  Columns 1 through 13

    0.9285    0.9564    0.8928    1.0096    0.9665    0.5975    0.5774    0.6000    0.6602    0.6021    0.9060    0.9270    0.8567
    0.1479    0.2455    0.2907    0.2293    0.2399    0.5223    0.5951    0.4917    0.4889    0.4444    0.6953    0.8433    0.8050

  Columns 14 through 20

    0.9540    0.8193    0.4365    0.4972    0.5210    0.5160    0.4555
    0.6824    0.7907    0.0530    0.0541    0.0519   -0.0416    0.0107

>> D = 0;
>> for n=1:20, d = sum((repmat(X(:,n),1,size(Y,2)) - Y).^2,1); k(n) = min(find(d==min(d))); D = D + min(d); end;
>> D = D/20

D =

    0.0899

>> k

k =

     4     4     4     4     4     1     1     1     1     1     3     1     1     3     1     2     2     2     2     2
```

5.2. Condição do Centróide

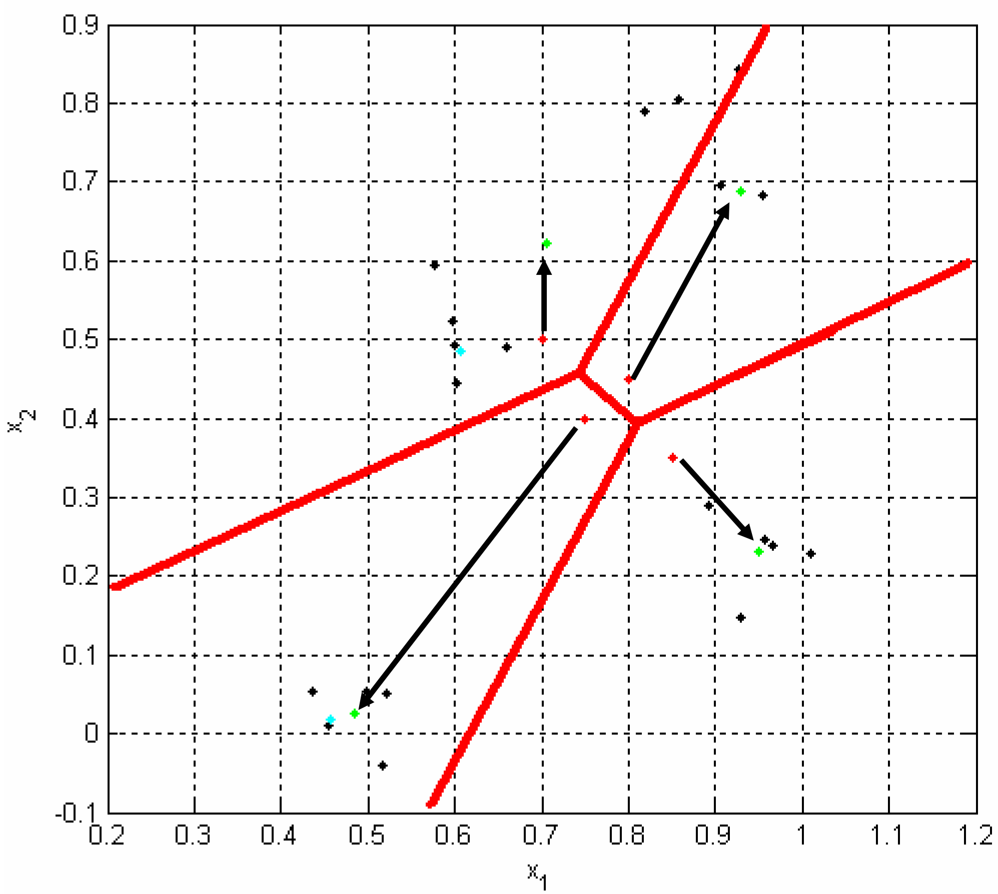
$$D_{min} = \frac{1}{N} \sum_{n=1}^{N} (\mathbf{x}(n) - \mathbf{y}_{k(n)})^T (\mathbf{x}(n) - \mathbf{y}_{k(n)})$$

$$D_{min} = \frac{1}{N} \sum_{n=1}^{N} (\mathbf{x}(n)^T \mathbf{x}(n) - \mathbf{x}(n)^T \mathbf{y}_{k(n)} - \mathbf{y}_{k(n)}^T \mathbf{x}(n) + \mathbf{y}_{k(n)}^T \mathbf{y}_{k(n)})$$

$$\frac{\partial D_{min}}{\partial \mathbf{y}_k} = \frac{2}{N} (N_k \mathbf{y}_k - \sum_{\mathbf{x}(n) \in R_k} \mathbf{x}(n)) = 0$$

$$\mathbf{y}_k = \frac{1}{N_k} \sum_{\mathbf{x}(n) \in R_k} \mathbf{x}(n)$$

5.2. Condição do Centróide



5.2. Condição do Centróide

```
>> k

k =

     4     4     4     4     4     1     1     1     1     1     3     1     1     3     1     2     2     2     2     2

>> p = zeros(K,1); Y = zeros(size(Y)); for n=1:20, Y(:,k(n)) = Y(:,k(n)) + X(:,n); p(k(n)) = p(k(n)) + 1; end;
>> p

p =

     8
     5
     2
     5

>> for j=1:K, Y(:,j) = Y(:,j)/p(j); end;
>> Y

Y =

    0.7050    0.4852    0.9300    0.9508
    0.6227    0.0256    0.6888    0.2306

>> plot(Y(1,:),Y(2,:),'g.');
>> D = 0; for n=1:20, D = D + sum((X(:,n)-Y(:,k(n))).^2);
end;
>> D = D/20

D =

    0.0178
```